Calculus Supplement

This appendix briefly reviews the material of the textbook in the language of calculus. There are sections corresponding to most of the textbook chapters, and most have some brief exercises at the end.

Chapter 1

1. Demand and Supply. Demand and supply are functions that convert prices to quantities. For a given price P, the demand and supply functions are defined by setting $D(P)$ = the quantity that demanders wish to buy at price P, and $S(P)$ = the quantity that suppliers wish to sell at price P.

When the price changes from P_1 to P_2, the quantity demanded changes from $D(P_1)$ to $D(P_2)$, but the function D remains unchanged. A *change in demand* refers to a change in the function D itself; similarly for supply.

Because the output from a demand or supply function represents a quantity, it is often denoted by the letter Q. Thus, if we are discussing demand we write

$$Q = D(P)$$

and if we are discussing supply we write

$$Q = S(P)$$

Of course, when we discuss supply and demand simultaneously, we cannot use the same symbol to denote the output from both functions. In that case, we usually write

$$Q_d = D(P)$$

$$Q_s = S(P)$$

to distinguish the quantity demanded Q_d from the quantity supplied Q_s.

2. Derivatives. The fact that the demand curve slopes downward is expressed by the inequality

$$D'(P) < 0$$

or

$$\frac{dQ_d}{dP} < 0$$

The fact that the supply curve slopes upward is expressed by the inequality

$$S'(P) > 0$$

or

$$\frac{dQ_s}{dP} > 0$$

3. *Equilibrium.* The equilibrium price is the price P at which $D(P) = S(P)$, and the equilibrium quantity is this common value. The assumptions $D'(P) < 0$ and $S'(P) > 0$ ensure the uniqueness of the equilibrium.

4. *Taxation.* Suppose that the demand for lettuce is given by the function $Qd = D_0(P)$. When a sales tax equal to T per head of lettuce is imposed, the demander must pay $P + T$ to acquire a head of lettuce. Thus, the new demand function is given by the formula $D_1(P) = D_0(P + T)$. The graph of D_1 is identical to the graph of D_0 translated downward a distance T.

Similarly, an excise tax of T per head of lettuce causes the supply function $S_0(P)$ to be replaced by the new function $S_1(P) = S_0(P + T)$. The graph of S_1 is identical to the graph of S_0 translated upward a distance T.

Continue to write D_0 and S_0 for the original demand and supply functions. A sales tax leads to the equilibrium price P_{sales}, where

$$D_1(P_{\text{sales}}) = S_0(P_{\text{sales}})$$

and an excise tax leads to the equilibrium price P_{excise}, where

$$D_0(P_{\text{excise}}) = S_1(P_{\text{excise}})$$

Substituting the expressions for D_1 and S_1 gives

$$D_0(P_{\text{sales}} + T) = S_0(P_{\text{sales}})$$

$$D_0(P_{\text{excise}}) = S_0(P_{\text{excise}} + T)$$

It is easy to check that if P_{sales} satisfies the first equation, then $P_{\text{excise}} = P_{\text{sales}} + T$ satisfies the second equation. Moreover, there is only one solution to each equation because D_0 is decreasing and S_0 is increasing. It follows that we must have $P_{\text{excise}} = P_{\text{sales}} + T$.

Under a sales tax, demanders pay $P_{\text{sales}} + T = P_{\text{excise}}$ and suppliers get P_{sales}. Under an excise tax, demanders pay P_{excise} and suppliers keep $P_{\text{excise}} - T = P_{\text{sales}}$. Therefore, the sales and excise taxes are equivalent.

Exercises

1. Let $S(P)$ and $D(P)$ be the supply and demand functions for apples. Suppose that an excise tax of T is imposed and the posttax equilibrium price for apples is P. Treating P as a function of T, use the equation

$$D(P) = S(P - T)$$

and the chain rule to derive a formula for the derivative dP/dT.

$$\textit{Answer: } \frac{dP}{dT} = \frac{S'(P - T)}{S'(P - T) - D'(P)}$$

2. In problem 1, let P_{sellers} be the price that sellers actually receive for the items they sell. Use the result of problem 1 and the equation

$$P_{\text{sellers}} = P - T$$

to derive and simplify a formula for the derivative dP_{sellers}/dT.

$$\text{Answer: } \frac{dP}{dT} = \frac{D'(P)}{S'(P-T) - D'(P)}$$

3. Let $S(P)$ and $D(P)$ be the supply and demand functions for apples. Suppose that a sales tax of T is imposed and the posttax equilibrium price for apples is P. Treating P as a function of T, use the equation

$$D(P + T) = S(P)$$

and the chain rule to derive a formula for the derivative dP/dT.

4. In problem 3, let P_{buyers} be the price that buyers actually pay for the items they buy. Use the result of problem 3 and the equation

$$P_{\text{buyers}} = P + T$$

to derive and simplify a formula for the derivative dP_{buyers}/dT.

5. Explain how your solutions to problems 1 through 4 illustrate the proposition that the economic incidence of a tax is independent of its legal incidence.

Chapter 3

1. Families of Indifference Curves. A single indifference curve is defined by a single equation in X and Y. A *family* of indifference curves must be defined by a *family* of equations. The easiest way to do this is to specify a function of two variables, $U(X, Y)$, and to consider the family of equations

$$U(X, Y) = C$$

where C is any constant. Thus, for example, one indifference curve is given by $U(X, Y) = 1$, another by $U(X, Y) = 2$, and so forth.

When the indifference curves are described in this way, it is clear that they fill the plane (for every (X, Y), $U(X, Y)$ has *some* value, and (X, Y) is on the corresponding indifference curve) and that they never cross (for every (X, Y) $U(X, Y)$ has only *one* value, and so (X, Y) is on only one indifference curve). The other properties of indifference curves follow from some assumptions on U. A set of assumptions sufficient to guarantee the desired properties is

$$\frac{\partial U}{\partial X} > 0 \tag{1}$$

$$\frac{\partial U}{\partial Y} > 0 \tag{2}$$

$$\frac{\partial^2 U}{\partial X^2} < 0 \tag{3}$$

$$\frac{\partial^2 U}{\partial Y^2} < 0 \tag{4}$$

$$\frac{\partial^2 U}{\partial X^2} \cdot \frac{\partial^2 U}{\partial Y^2} - \left(\frac{\partial^2 U}{\partial X\,\partial Y}\right)^2 > 0 \tag{5}$$

2. Properties of Indifference Curves. To see what our assumptions imply about the indifference curves, fix a constant C and look at the indifference curve defined by $U(X, Y) = C$. This curve is also the graph of the function $Y = f(X)$, where f is implicitly defined by the formula

$$U(X,f(X)) = C$$

The chain rule gives

$$\frac{\partial U}{\partial X}(X,f(X)) + \frac{\partial U}{\partial Y}(X,f(X)) \cdot \frac{df}{dX}(X) = \frac{dC}{dX} = 0$$

so that

$$\frac{df}{dX} = -\frac{\partial U/\partial X}{\partial U/\partial Y} < 0$$

(The final inequality follows from assumptions (1) and (2).) In other words, indifference curves slope downward.

By differentiating both sides of the formula

$$\frac{\partial U}{\partial X}(X,f(X)) + \frac{\partial U}{\partial Y}(X,f(X)) \cdot \frac{df}{dX}(X) = 0$$

we find that

$$\frac{\partial^2 U}{\partial X^2}(X,f(X)) + 2 \cdot \frac{\partial^2 U}{\partial X\,\partial Y}(X,f(X)) \cdot \frac{df}{dX}(X) + \frac{\partial^2 U}{\partial Y^2} \cdot \left(\frac{df}{dX}(X)\right)^2$$
$$= \frac{\partial U}{\partial Y}(X,f(X)) \cdot \frac{d^2 f}{dX^2}(X)$$

For a given value of X, the left side of this equation is equal to the value of the quadratic function

$$(t) = \frac{\partial^2 U}{\partial X^2}(X,f(X)) + 2 \cdot \frac{\partial^2 U}{\partial X\,\partial Y}(X,f(X)) \cdot t + \frac{\partial^2 U}{\partial Y^2}(X,f(X)) \cdot t^2$$

at the point $t = (df/dX)(X)$. Assumptions (3) and (5) imply that the quadratic function takes only negative values.* Thus, we know that $-(\partial U/\partial Y)(X, f(X))\,(\partial^2 f/\partial X^2)(X)$ is negative. Together with assumption 1 this allows us to conclude that $(d^2 f/dX^2)$ is everywhere positive. In other words, indifference curves are convex.

To summarize, we have shown that when the indifference curves are described by the formulas $U(X, Y) = C$, and when U satisfies assumptions (1) through (5), we may conclude that indifference curves fill the plane, never cross, slope downward, and are convex.

3. The Marginal Rate of Substitution and the Consumer's Optimum. According to the chain rule, the slope of the indifference curve at the point (X, Y) is given by

$$-\frac{\partial U/\partial X}{\partial U/\partial Y}$$

* Assumption (3) implies that $Q(0) < 0$. Assumption (5) implies that Q has no real roots. By the intermediate value theorem, a continuous function that takes one negative value and has no real roots must take only negative values.

evaluated at (X, Y). The absolute value of this slope is the marginal rate of substitution between X and Y.

The consumer's budget constraint is given by the equation

$$P_X \cdot X + P_Y \cdot Y = 1$$

where P_X, P_Y, and I are constants. Its graph is the equation of the straight line through $(0, I/P_Y)$ and $(I/P_X, 0)$; the slope of this line is $-P_X/P_Y$.

In order to attain the highest possible indifference curve, the consumer maximizes $U(X, Y)$ subject to the budget constraint. There are two ways to solve this problem. One is to view it as a constrained maximization problem in the two variables X and Y so that the method of Lagrange multipliers applies. However, there is a much more elementary alternative. Using the budget constraint, we solve for Y and get

$$Y = \frac{1}{P_Y} - \frac{P_X}{P_Y} \cdot X$$

Then we are reduced to solving a maximization problem in one variable; namely, maximize

$$U\left(X, \frac{1}{P_Y} - \frac{P_X}{P_Y} \cdot X\right)$$

The first-order condition is

$$\frac{\partial U}{\partial X} = \frac{P_X}{P_Y} \cdot \frac{\partial U}{\partial Y}$$

or

$$\frac{\partial U/\partial X}{\partial U/\partial Y} = \frac{P_X}{P_Y}$$

That is, the consumer selects the point on the budget constraint at which the marginal rate of substitution and the relative price of X are equal. This has good intuitive content, as described in the textbook.

To verify that we have found a maximum, it is necessary to verify the second-order condition as well. Although it is geometrically obvious that we have indeed found a maximum (see, for example Exhibit 3.9 in the textbook), you might want to verify the second-order condition directly, using assumptions (1) through (5).

Exercises

1. Suppose that your indifference curves between X and Y are given by the family of equations $U(X, Y) = C$, where $U(X, Y) = X^{1/2} \cdot Y^{1/2}$.

- **a.** Does U satisfy the conditions 1 through 5 of section 1?

  Answer: Yes.

- **b.** Compute the slope of your indifference curve passing through the point (X, Y) at that point, as a function of X and Y.

  Answer: Y/X.

- **c.** Show that your indifference curves are convex.

d. Suppose that the price of X is \$1, the price of Y is \$2, and your income is \$10. What basket of goods do you buy?

Answer: $X = 5$, $Y = 2\frac{1}{2}$.

e. Repeat part (d) if the price of X goes up to \$5.

Answer: $X = 1$, $Y = 2\frac{1}{2}$.

f. Repeat part (d) if your income goes up to \$20.

2. Repeat problem 1 with the function U replaced by $U(X, Y) = X^{1/4} \cdot Y^{1/4}$.

Chapter 4

1. The Engel Curve. We will continue to assume that the consumer's indifference curves are the curves $U(X, Y) = C$ for some fixed function U.

In order to see how the consumer reacts to changes in income, we hold the prices of X and Y fixed; that is, we treat P_X and P_Y as constants. We can always choose to measure Y in units that make $P_Y = 1$ (for example, if Y is Coca-Cola and it sells for 50¢ per can, then we will declare one "unit" of Coca-Cola to consist of two cans). This allows us to adopt the abbreviation $P = PX$; that is, P is the relative price of X in terms of Y.

At any given level of income I, the consumer decides what quantity of X to purchase. We will denote this quantity by $E(I)$. $E(I)$ is chosen to maximize

$$U(E(I), I - PE(I))$$

That is, $E(I)$ satisfies the first-order condition

$$U_1(E(I), I - PE(I)) = PU_2(E(I), I - PE(I)) \tag{6}$$

where we have abbreviated

$$U_1 = \frac{\partial U}{\partial X} \qquad U_2 = \frac{\partial U}{\partial Y}$$

The function $E(I)$ implicitly defined by equation (6) is the consumer's Engel curve for X.

By differentiating equation (6) with respect to I, we can find the slope of the Engel curve. You should verify that

$$E'(I) = \frac{-U_{12} + PU_{22}}{U_{11} - 2PU_{12} - P^2U_{22}} \tag{7}$$

As an example, suppose that there are constants α and β such that

$$U(X, Y) = X^{\alpha} Y^{\beta}$$

Then you should be able to verify that the equation of the Engel curve is given by

$$E(I) = \frac{\alpha I}{(\alpha + \beta)P}$$

That is, in this case the consumer's Engel curve is a straight line through the origin with slope $\alpha/[(\alpha + \beta) \cdot P]$.

2. The Demand Curve. The consumer's demand curve D is derived in a similar way, by treating I as a constant and noting that the consumer maximizes $U(X, I - PX)$ by setting $X = D(P)$, where $D(P)$ satisfies

$$U(D(P), I - PD(P)) = PU_2(D(P), I - PD(P)) \qquad \textbf{(8)}$$

The function D implicitly defined by equation (8) is the consumer's demand curve. For example, if the indifference curves are given by

$$U(X, Y) = X^\alpha Y^\beta$$

then the demand curve for X is given by

$$D(P) = \frac{\alpha I}{(\alpha + \beta)P}$$

Although the right-hand expression looks exactly like the expression for the Engel curve, we are now treating P as the independent variable and I as a constant. Thus, the demand curve in this case is a hyperbola.

By differentiating equation (8) with respect to P, we can find the slope of the demand curve. You should verify that

$$D'(P) = \frac{U_{12}D(P) - PU_{22}D(P) + U_2}{U_{11} - 2PU_{12} + P^2U_{22}} \qquad \textbf{(9)}$$

3. The Compensated Demand Curve. We can also derive an expression for the compensated demand curve $Dc(X)$. Suppose the consumer starts out on the indifference curve $U(X, Y) = C$. In order to derive the compensated demand curve, we pretend that regardless of how the price P changes, the consumer is constrained to remain on the same indifference curve. Thus, for any price P, the consumer selects quantities $X = Dc(P)$ and $Y = f(P)$ such that

$$U(D_c(P), f(P)) = C \qquad \textbf{(10)}$$

Differentiating this with respect to P, we find that

$$f'(P) = \frac{U_1 D'_c}{U_2} = PD'_c \qquad \textbf{(11)}$$

(The last equality results from the fact that the consumer still maximizes by setting $U_1 = PU_2$.)

From the fact that the consumer is maximizing subject to the price P, we have

$$U_1(D_c(P), f(P)) = PU_2(D_c(P), f(P)) \qquad \textbf{(12)}$$

The function D_c is defined implicitly by this together with equation (10). Differentiating equation (12) with respect to P and substituting for $f'(P)$ as per equation (11), we get

$$D'_c = \frac{U_2}{U_{11} - 2PU_{12} + P^2U_{22}} \qquad \textbf{(13)}$$

We have noted earlier that the denominator in this expression must be negative in consequence of equations (3) and (5), and the numerator is positive by equation (2). It follows that $D_c'(P)$ is unambiguously negative; the compensated demand curve must be downward-sloping.

4. Substitution and Income Effects. There is an interesting consequence of equations (7), (9), and (13). Combining them, we find that for any given P and I, we have

$$D'(P) = D'_c(P) - D(P)E'(I) \tag{14}$$

(In interpreting this equation, keep in mind that the functions D and D_c depend on I and that the function E depends on P.) This says that when P changes, the corresponding change in quantity demanded can be decomposed into two parts: first a movement along the compensated demand curve (the substitution effect) and then an additional movement whose size depends on the slope of the Engel curve (the income effect). If the Engel curve is upward sloping (that is, if X is a normal good), then equation (14) shows that both components are negative—the income effect reinforces the substitution effect, so the demand curve must slope downward. If the Engel curve is downward sloping, then $D'(P)$ has one negative component and one positive component—the income effect works counter to the substitution effect. In this case, it is at least theoretically possible for the demand curve to slope upward—the case of a Giffen good.

5. Elasticities. The income elasticity of demand for a commodity is

$$\frac{I}{Q} \cdot \frac{dQ}{dI}$$

where dQ/dI is the derivative of the Engel curve, calculated in expression (7). An equivalent expression is

$$\frac{d(\log Q)}{d(\log I)}$$

Likewise, we define the price elasticity of demand to be

$$\frac{P}{Q} \cdot \frac{dQ}{dP}$$

where dQ/dP is the derivative of the demand function calculated in expression (9). An equivalent expression is

$$\frac{d(\log Q)}{d(\log P)}$$

The compensated price elasticity of demand is

$$\frac{P}{Q_c} \cdot \frac{dQ_c}{dP}$$

where dQ_c/dP is the derivative of the compensated demand function calculated in expression (13). An equivalent expression is

$$\frac{d(\log Q_c)}{d(\log P)}$$

6. The Slutsky Equation* If we multiply equation (14) through by $P/D(P) = P/E(I)$, we get

$$\begin{pmatrix}\text{Elasticity of the}\\ \text{ordinary demand curve}\end{pmatrix} = \begin{pmatrix}\text{Elasticity of the}\\ \text{compensated demand curve}\end{pmatrix} + P \cdot E'(I)$$

* This is a topic not covered in the body of the textbook.

The last term on the right can be rewritten as

$$\frac{P \cdot E}{I} \cdot \begin{pmatrix} \text{Elasticity of the} \\ \text{Engel curve} \end{pmatrix}$$

and $\frac{P \cdot E}{I}$ can be interpreted as the fraction of his income that the consumer spends on X. Thus, we have

$$\begin{pmatrix} \text{Elasticity of the} \\ \text{ordinary demand curve} \end{pmatrix} = \begin{pmatrix} \text{Elasticity of the} \\ \text{compensated demand curve} \end{pmatrix} - \begin{pmatrix} \text{Fraction of income} \\ \text{spent on } X \end{pmatrix} \cdot \begin{pmatrix} \text{Elasticity of the} \\ \text{Engel curve} \end{pmatrix}$$

The preceding equation is called the *Slutsky equation*. It shows, for example, that if the fraction of his income that the consumer spends on X is small, then the elasticities of the ordinary and compensated demand curves are approximately equal.

Exercises

1. Suppose that indifference curves are given by the family of equations $U(X, Y) = X^{1/2} \cdot Y^{1/2} = C$, the price of X is \$1, the price of Y is \$2, and income is \$10. One day the price of X goes up to \$2. What happens to consumption of X? How much of this change is due to the substitution effect and how much is due to the income effect?

Answer: Consumption falls from 5 to 21/2. The fall from 5 to $\sqrt{7.5} \approx$ is the substitution effect and the remainder is the income effect.

2. Repeat problem 1 with the function U replaced by

$$V'(x) - C'(x) = C$$

Chapter 5

1. A Farmer's Problem. Consider a farmer who must decide how many acres of land to spray for insects. If he sprays x acres, the value of the crops saved is given by the function $V(x)$. The rate at which V grows as additional acres are sprayed is given by the derivative $V'(x)$, which we call the *marginal value* of the crops saved, or the *marginal benefit* from spraying. In general, the word *marginal* in economics refers to a first derivative.

When one acre is a small part of the total area under consideration, $V'(x)$ can be well approximated by the quantity $V(x) - V(x - 1)$. The latter expression is used as the definition of marginal value in the textbook, but the more precise definition is $V'(x)$.

Suppose that the cost of spraying x acres is given by the function $C(x)$. The farmer's goal is to maximize the quantity $V(x) - C(x)$, which he accomplishes by setting

$$V'(x) - C'(x) = C$$

In other words, he sets

$$V'(x) = C'(x)$$

Or, in still other words, he chooses that quantity at which marginal benefit is equal to marginal cost.

If a constant is added to the function C, then that same constant is subtracted from the function $V - C$. The addition or subtraction of a constant cannot change the location of the maximum, and therefore the number of acres sprayed will not change. Put another way, the addition of the constant does not change the derivative C' and hence the quantity at which $V' = C'$ does not change.

Of course, the function C can change in many ways other than by the addition of a constant, and in general other such changes in C will affect the farmer's actions.

2. Firms and Profit Maximization. A firm seeks to maximize its profits, which are defined as revenues minus costs. The firm must select a quantity of output to produce. Let us denote the total revenue derived from producing and selling Q units of output by $TR(Q)$ and the total cost of producing and selling Q units of output by $TC(Q)$. Let $D(P)$ be the demand curve for the firm's product. Then since $D^{-1}(Q)$ is the maximum price at which the firm can sell Q units of output, it follows that

$$TR(Q) = Q \cdot D^{-1}(Q)$$

The firm seeks to maximize

$$TR(Q) - TC(Q)$$

which it accomplishes by selecting the quantity Q, at which

$$TR'(Q) - TC'(Q) = 0$$

or

$$TR'(Q) = TC'(Q)$$

If the TC function changes by the addition of a constant, then the derivative TC' is unchanged and consequently so is the profit-maximizing quantity. Put another way, the addition of a constant to TC simply subtracts a constant from the profit function $TR - TC$, and the subtraction of a constant cannot change the location of the maximum.

Other sorts of changes in TC can change the optimal output level, as can changes in TR. Since we have already seen that $TR(Q) = Q \cdot D^{-1}(Q)$, it follows that any change in TR must arise from a change in the demand function D.

Exercises

1. A firm faces the demand function $D(P) = 100 - 2P$ and the total cost function $TC(Q) = Q^2$. How much does it produce and at what price?

   Answer: $Q = 16\frac{2}{3}$, $P = 41\frac{2}{3}$.

2. A firm faces the demand function $D(P) = P^{-1/2}$ and the total cost function $TC(Q) = Q^2$. How much does it produce and at what price?

Chapter 6

1. Short-Run Costs. In the short run, we take the firm's capital usage to be fixed at some quantity, so that total product TP is a function only of labor L. The marginal product of labor is $MP(L) = TP'(L)$.

To find the short-run total cost of producing Q units of output, note that it is necessary to employ $TP^{-1}(Q)$ units of labor so that the total cost of production is

$$P_K \cdot K_0 + P_L \cdot TP^{-1}(Q)$$

where P_K and P_L are the hire prices of capital and labor. Differentiating this total cost function, we find that the firm's short-run marginal cost curve is given by

$$MC(Q) = \frac{1}{MP_L(L)}$$

where L is the quantity of labor used in the production of Q units of output. We define the firm's *variable cost* (VC) to be $P_L \cdot L$, its *average cost* (AC') to be TC/Q, and its *average variable cost* (AVC) to be VC/Q. To find the relations among these cost curves, note for example that

$$\begin{aligned}\frac{dAC}{dQ} &= \frac{d(TC/Q)}{dQ}\\ &= \frac{Q\dfrac{dTC}{dQ} - TC}{Q^2}\\ &= \frac{MC}{Q} - \frac{AC}{Q}\end{aligned}$$

From this we conclude that when AC is minimized (so that dAC/dQ is zero), we must have $MC/Q = AC/Q$, or equivalently, $MC = AC$. In other words, the bottom of the U-shaped average cost curve occurs where MC crosses AC. A similar calculation holds with AC replaced by AVC.

The same equation shows that when MC is below AC, dAC/dQ is negative, so that AC is downward sloping, and when MC is above AC, dAC/dQ is positive, so that AC is upward sloping.

2. Isoquants and the Production Function. The technology available to a firm is specified by its *production function* $f(L, K)$, which tells how much output the firm can produce using L units of labor and K units of capital. We assume that the production function satisfies the analogues of properties (1) through (5), which were assumed for the utility function. The isoquants are then the graphs of the various curves $f(L, K) = C$, where C is any constant.

Along the isoquant $f(L, K) = C$, K is implicitly defined as a function $g(L)$. Using the chain rule to differentiate both sides of the formula

$$f(L, g(L)) = C$$

we find that the slope of the isoquant is

$$g'(L) = \frac{\partial f/\partial L}{\partial f/\partial K} \tag{15}$$

As we will see in the next paragraph, $\partial f/\partial L$ and $\partial f/\partial K$ can be interpreted as the marginal products of labor and of capital.

3. Long-Run Costs. In the long run, both capital and labor can be varied. The firm seeks to maximize the output that it can produce at any given cost. For a given expenditure E, the firm can hire any basket of inputs (L, K) such that

$$P_L \cdot L + P_K \cdot K = E$$

Let us rewrite this as

$$K = \frac{E}{P_K} - \frac{P_L \cdot L}{P_K}$$

Then the firm's problem is to maximize

$$f\left(L, \frac{E}{P_K} - \frac{P_L \cdot L}{P_K}\right)$$

Differentiating with respect to L, we find that the firm chooses those quantities L of labor and $K = (E - P_L \cdot L)/P_K$ of capital at which

$$f_1(L, K) - \frac{P_L}{P_K} \cdot f_2(L, K) = 0$$

or equivalently

$$\frac{f_1(L, K)}{f_2(L, K)} = \frac{P_L}{P_K} \tag{16}$$

That is, the firm chooses an input mix at which the ratio of the marginal products is equal to the ratio of the input prices. Because the ratio of the marginal products is the absolute slope of the isoquant (that is, it is the marginal rate of technical substitution), and because the input price ratio is the absolute slope of the isocosts, it follows that the firm operates at a tangency between an isoquant and an isocost. There are many such tangencies, one for each level of expenditure. The curve formed by these tangencies is the *expansion path.* The expansion path is the graph of equation (16).

For an alternative viewpoint, we can envision the firm minimizing cost for any given level of output. Thus, if $K = g(L)$ is the equation of the isoquant corresponding to the given output, the firm's problem is to minimize

$$P_L \cdot L + P_K \cdot g(L)$$

Differentiating, we find that the firm operates where $g'(L) = -P_L/P_K$; in view of equation (15) this is the same condition as described by equation (16).

For a given quantity of output Q, let $L_0(Q)$ and $K_0(Q)$ be the quantities of inputs that the firm employs in order to produce Q units at the lowest possible cost. Then the functions L_0 and K_0 are determined implicitly by the equations

$$\frac{f_1(L_0(Q), K_0(Q))}{f_2(L_0(Q), K_0(Q))} = \frac{P_L}{P_K}$$

$$f(L_0(Q), K_0(Q)) = Q$$

The first of these equations says that the firm is on its expansion path, and the second says that it produces quantity Q. Differentiating the second equation yields

$$f_1\,(L_0(Q), K_0(Q)) \cdot L'_0(Q) + f_2\,(L_0(Q), K_0(Q)) \cdot K'_0(Q) = 1$$

Combining this with first equation gives

$$f(L_0(Q), K_0(Q)) \cdot \left(L'_0(Q) + \frac{P_K}{P_L} \cdot K'_0(Q)\right) = 1 \tag{17}$$

The long-run total cost of producing Q units of output is

$$LRTC(Q) = P_L \cdot L_0(Q) + P_K \cdot K_0(Q)$$

Thus, the long-run marginal cost is given by

$$LRMC(Q) = P_L \cdot L'_0(Q) + P_K \cdot K'_0(Q)$$
$$= P_L \cdot \left(L'_0(Q) + \frac{P_K}{P_L} \cdot K'_0(Q) \right)$$
$$= \frac{P_L}{f_1\,(L_0(Q), K_0(Q))} \quad \textbf{(18)}$$

(The last equality follows from equation (17).)

A similar calculation shows that we also have

$$LRMC(Q) = \frac{P_K}{f_2\,(L_0(Q), K_0(Q))}$$

Here is a slightly different way to view the long-run total cost curve: For each quantity of capital K, let $SRTC_K(Q)$ be the short-run total cost curve that results when the firm uses K units of capital. In the long run, the firm chooses K to minimize its costs, so

$$LRTC(Q) = min_K SRTC_K(Q)$$

Thus, the long-run total cost curve lies below all of the short-run total cost curves.

4. Returns to Scale. For any given L and K, define

$$\Gamma = \frac{L \cdot f_1(L, K) + K \cdot f_2(L, K)}{f(L, K)}$$

Then we say that the production function f exhibits constant, decreasing, or increasing *returns to scale* at (L, K) according to whether Γ is equal to, less than, or greater than 1.

Suppose that both inputs are increased by the same proportion h, so that the new quantities of labor and capital are $(1 + h)L$ and $(1 + h)K$. Then for h small we have

$$f((1 + h)L, (1 + h)K) = f(L + hL, K + hK)$$
$$= f(L, K) + hLf_1(L, K) + hKf_2(L, K)$$
$$= (1 + \Gamma \cdot h) \cdot f(L, K)$$

In other words, the proportional change in output is equal to, less than, or greater than the proportional change in the inputs depending on whether f exhibits constant, decreasing, or increasing returns to scale. This is the definition given in the textbook.

A case of particular interest is that of a *homogeneous* production function. A homogeneous production function is defined to be one for which Γ is a constant independent of K and L. In this case, we say that Γ is the *degree of homogeneity* of the function f, or that f is *homogeneous of degree* Γ. As an immediate consequence of the definition in the textbook, a homogeneous function of degree 1 exhibits constant returns to scale, and a homogeneous function of degree less than (greater than) 1 exhibits decreasing (increasing) returns to scale.

5. Returns to Scale and the Long-Run Average Cost Curve. We can relate the returns to scale to the slope of the long-run average cost curve. The slope of the long-run average cost curve is

$$\frac{dLRAC}{dQ} = \frac{d(LRTC/Q)}{dQ} = \frac{(dLRTC/dQ) \cdot Q - LRTC}{Q_2} = \frac{(LRMC - LRAC)}{Q}$$

Thus, long-run average cost is flat, increasing, or decreasing depending on whether *LRAC* is equal to, less than, or greater than *LRMC*. To investigate this, we consider the ratio *LRAC/LRMC*. We have

$$\frac{LRAC(Q)}{LRMC(Q)} = \frac{(P_L \cdot L_0(Q) + P_K \cdot K_0(Q))}{f(L_0(Q), K_0(Q))} \Bigg| \frac{P_L}{f(L_0(Q), K_0(Q))} \quad \textbf{(by (18))}$$

$$= \frac{L \cdot f_1(L_0(Q), K_0(Q)) + f_2(L_0(Q), K_0(Q))}{f(L_0(Q), K_0(Q))} \quad \textbf{(by (16))}$$

Since the final term in the right-hand series of equations is none other than Γ, we see that

$$\text{When } \Gamma \text{ is} \begin{Bmatrix} \text{equal to} \\ \text{less than} \\ \text{greater than} \end{Bmatrix} 1, \, LRAC \text{ is} \begin{Bmatrix} \text{equal to} \\ \text{less than} \\ \text{greater than} \end{Bmatrix} LRMC \text{ and therefore} \begin{Bmatrix} \text{flat} \\ \text{increasing} \\ \text{decreasing} \end{Bmatrix}$$

In other words, constant returns to scale imply a flat *LRAC*, decreasing returns to scale imply an increasing *LRAC*, and increasing returns to scale imply a decreasing *LRAC*.

6. Relations between the Short Run and the Long Run. Given the long-run production function $f(L, K)$, and given a fixed quantity of capital K_0, we derive the short-run production function

$$TP(L) = f(L, K_0)$$

Thus, the marginal product of labor is given by

$$MP(L) = TP'(L) = \frac{\partial f}{\partial L}(L, K_0)$$

Let $C(Q, K)$ be the cost of producing Q units of output using K units of capital (together with however much labor is necessary). Thus, for fixed K, $SKTC(Q) = C(Q, K)$ is the short-run total cost curve, and short-run marginal cost is given by

$$SRMC(Q) = \frac{\partial C}{\partial Q}(Q, K)$$

Now for any given Q let $K_0(Q)$ be the quantity of capital that allows Q units of output to be produced at the lowest cost. Then $LRTC(Q) = C(Q, K_0(Q))$ is the long-run total cost curve, and long-run marginal cost is given by

$$LRMC(Q) = \frac{\partial C}{\partial Q}(Q, K_0(Q)) + \frac{\partial C}{\partial K}(Q, K_0(Q)) \cdot K'_0(Q)$$

Since $K_0(Q)$ is determined by the first-order condition

$$\frac{\partial C}{\partial K}(Q, K_0(Q)) = 0$$

it follows that in long-run equilibrium (where $K = K_0(Q)$), we have

$$SRMC(Q) = LRMC(Q)$$

Interpreting marginal cost as the slope of total cost, this tells us that the short-run and long-run total cost curves are tangent where they touch. A similar argument applies to the short-run and long-run average cost curves.

Exercises

1. Suppose that a firm's production function is given by $f(L, K) = L^\alpha K^\beta$, where α and β are positive constants and both α and β are less than 1. When $K = 1$, write down the firm's (short-run) total product and marginal product of labor functions and its short-run marginal cost function, assuming that the wage rate of labor is 1. Repeat with $K = 2$. Does the firm experience diminishing marginal returns to labor?

   Answer: With $K = 1$, $TP_L = L^\alpha$, $MP_L = \alpha L^{\alpha - 1}$, and $MC = \dfrac{1}{\alpha L^{\alpha - 1}}$.

2. In problem 1, write down the equations for the 1-unit and 2-unit isoquant.

   Answer: The 1-unit isoquant is $L^\alpha K^\beta = 1$.

3. When the price of labor is W and the price of capital is R, what combination of inputs does the firm in problem 1 use to produce 1 unit of output? 2 units of output? Q units of output?

   Answer: For 1 unit of output,

$$L = \left(\frac{\alpha R}{\beta W}\right)^{\beta/\beta + \alpha} \text{ and } K = \left(\frac{\beta W}{\alpha R}\right)^{\alpha/\beta + \alpha}$$

4. In problem 1, write down the equations for the firm's long-run total cost and marginal cost curves.

   Answer: $LRTC(Q) = (CR^\beta W^\alpha Q)^{1/\alpha + \beta}$ where $C = \left[\left(\frac{\alpha}{\beta}\right)^\beta + \left(\frac{\beta}{\alpha}\right)^\alpha\right]$

5. In problem 1, suppose that $\alpha + \beta < 1$. Does the production function exhibit decreasing, constant, or increasing returns to scale? Repeat under the assumption that $\alpha + \beta = 1$ and then under the assumption that $\alpha + \beta > 1$.

6. Repeat problems 1 through 5 with the production function replaced by

$$f(L, K) = (L^\alpha + K^\alpha)^{\beta/\alpha}$$

Chapter 7

1. The Competitive Firm. A competitive firm is one that takes prices as given; that is, its own actions do not affect the market price of its product. For a competitive firm, total revenue is given by the simple formula $TR(Q) = P \cdot Q$, so that marginal revenue is the constant function $MR(Q) = P$.

For a competitive firm, the profit-maximizing rule $MC = MR$ simplifies to $MC = P$. That is, the firm produces that quantity Q for which $MC(Q) = P$. The exception is that there are some circumstances in which the firm might choose to shut down. It is shown in the text that the firm shuts down precisely if $P < AVC$. Thus, the competitive firm's supply curve is completely specified by the equation

$$S(P) = \begin{cases} MC^{-1}(\mathrm{P}) & \text{if P} \geq \min(AVC) \\ 0 & \text{if P} < \min(AVC) \end{cases}$$

This can be interpreted as a description of either the firm's short-run or long-run supply curve. To get the short-run cost curve, use the short-run marginal and average variable cost curves. To get the long-run cost curve, use the long-run marginal and average variable cost curves, keeping in mind that in the long run, average variable cost is just the same as average cost.*

2. The Competitive Industry in the Short Run. In the short run, we take the number of firms in the industry as given. To a first approximation, the industry supply curve is the sum of the individual firms' supply curves. To derive the industry supply curve precisely, it is necessary to take account of the factor-price effect, as discussed in the textbook.

Suppose that there are N firms in the industry, and that the i^{th} firm has the total cost function TC_i. Let Q be the total output of the entire industry. For each of the firms 2, . . ., n, let Q_i be the output of firm i, so that firm 1 produces the quantity

$$Q_1 = Q - \sum_{i=2}^{n} Q_i$$

Then a planner who wanted to minimize the sum of all firms' costs in producing Q units of output would choose the Q_i to minimize the expression

$$TC_1\left(Q - \sum_{i=2}^{n} Q_i\right) + \sum_{i=2}^{n} TC_i(Q_i)$$

Differentiating with respect to Q_i, we see that this requires setting $MC_1(Q_1) = MC_1(Q_1)$ for each i; that is, $MC_1(Q_i)$ must be independent of i. This condition is satisfied automatically in competitive equilibrium, because the i^{th} firm sets $MC_i(Q_i) = P$, and P is independent of i. A competitive industry minimizes the total cost of producing a given quantity.

3. The Competitive Industry in the Long Run. In the long run, we assume that there is free entry to the industry. The industry's long-run supply curve reflects this free entry. At any given price, we assume that sufficiently many firms enter to drive profits to zero, and the long-run supply curve shows the quantity that will be produced by that number of firms at the given price. The textbook discusses the various situations in which this could lead to a flat, increasing, or decreasing industry supply curve.

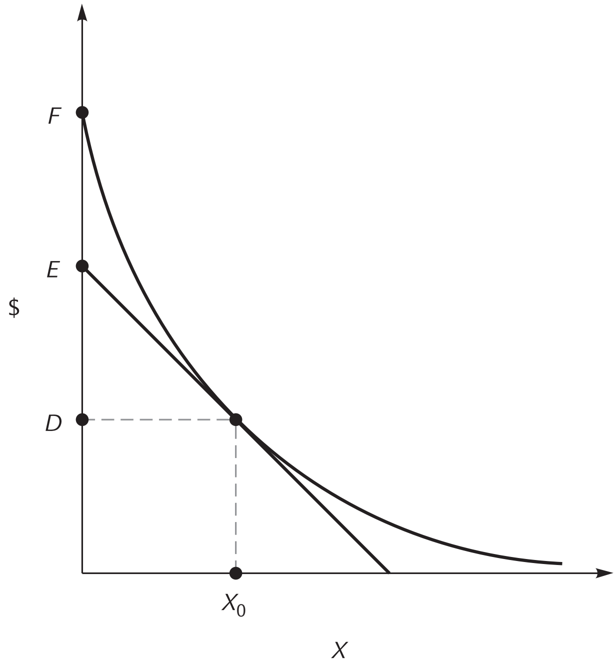
Exercise

1. Work the Numerical Exercises at the end of Chapter 7 in the body of the textbook.

* It is standard to assume that there are no fixed costs in the long run, since in the long run all factors of production are variable. Thus, there are no fixed costs as long as the firm's only costs are factor payments. It is possible to imagine some costs—such as annual license fees—that are fixed even in the long run. In this case, the average variable cost curve differs from the average cost curve, and it is average cost, not average variable cost, that determines whether the firm will remain in the industry.

Chapter 8

1. The Consumer's Surplus. Consider a consumer who has an income of \$$E$ and can purchase good X in the marketplace at a going price of \$$P$ per unit. He will choose to purchase the quantity x_0 depicted in the following diagram:



By enabling him to reach the illustrated indifference curve, the existence of the market has made the consumer as well off as if his income had increased to \$$F$. We say that he has earned a *consumer's surplus* of \$$(F - E)$. The equation of the indifference curve can be put in the form $Y = f(X)$. Then the consumer's compensated demand curve is given by the function

$$D_c(P) = (f')^{-1}(-P)$$

The inverse function is

$$D_c^{-1}(X) = -f'(X)$$

The area under the demand curve is the integral of the *inverse* function, because X is the variable on the horizontal axis. Therefore, the area under the compensated demand curve out to the quantity x_0 is given by

$$\int_0^{x_0} f'(X)dX = f(0) - f(x_0) = \$\,(F - D)$$

This is the total value to the consumer of x_0 units of X, in the sense that if all x_0 units were taken from the consumer and replaced by \$$(F - D)$, the consumer would remain on the same indifference curve.

When the consumer starts with \$$E$ and then trades for the optimal basket O, his total expenditure on good X is \$$(E - D)$. When this is subtracted from the area under the demand curve, we find that the remaining area (that is, the area under the demand curve and out to the quantity x_0, down to the price P) is

$$\$(F - D) - \$(E - D) = \$(F - E)$$

which is precisely the consumer's surplus.*

2. The Producer's Surplus. The producer's surplus is the excess of his revenue over his variable costs. Because

$$\mathrm{VC}(Q) = \int_0^Q MC(x)dx$$

it follows that the producer's surplus is the area above the marginal cost curve, out to the quantity supplied and up to the market price, as discussed in the textbook.

3. The Invisible Hand. Imagine a benevolent planner interested in the welfare of both consumers and producers. Suppose that the planner's goal is to maximize the total welfare gains earned in the market for X. That is, the planner wishes to maximize

$$TV(X) - TC(X)$$

where TV represents total value to consumers and TC represents total cost to producers. We have seen that when consumers purchase x_0 units of X, the total value of their purchases is

$$\int_0^{x_0} D_c(X)dX$$

Thus, the planner seeks to maximize

$$\int_0^{x_0} D_c(X)dX - TC(x_0)$$

Differentiating, we find that the optimum occurs where

$$D_c(x_0) = MC(x_0)$$

or, in other words, at the point where the demand and supply curves cross. This, of course, is none other than the point of equilibrium. The competitive equilibrium outcome is precisely the outcome sought by the planner.

4. General Equilibrium. Consider the Edgeworth box economy described in the text. There are two individuals (Aline and Bob) and two goods (Food and Clothing). Suppose that Aline's indifference curves are given by the family of equations $U(X, Y) = C$ and that Bob's are given by the family of equations $V(X, Y) = C$, where X is the quantity of food, Y is the quantity of clothing, and C varies over all possible constants.

We assume that the quantities of food and clothing are permanently fixed at X_0 and Y_0.

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* When we computed the total value to the consumer of being able to purchase good X, we assumed that the consumer is on the illustrated indifference curve, which is to say that we assumed that the market for good X actually does exist. Our measure of total value is the amount that the consumer would be willing to pay in order to prevent the market from disappearing. The resulting measure of consumer's surplus is called the *equivalent variation.* An alternative approach is to assume that the market for good X does not exist and to ask how much the consumer would be willing to pay to have the market come into existence. In this case, we would assume that the consumer is on the indifference curve through point E and integrate the corresponding compensated demand curve. The measure of consumer's surplus that arises in this way is called the *compensating variation.*

We will write X and Y for the quantities of food and clothing owned by Aline, so that $X_0 - X$ and $Y_0 - Y$ are the quantities owned by Bob. An *allocation* is a specification of Aline's basket (X, Y) (which then determines Bob's basket as well). An allocation (X, Y) is *Pareto-optimal* if no other allocation could make both Aline and Bob better off; that is, (X, Y) is Pareto optimal if there does not exist any allocation (X', Y') such that $U(X', Y') > U(X, Y)$ and $V(X_0 - X', Y_0 + Y') > V(X_0 + X, Y_0 + Y)$.*

We can show that the allocation (X, Y) is Pareto-optimal if and only if

$$\frac{\partial U/\partial X}{\partial U/\partial Y}(X, Y) = \frac{\partial V/\partial X}{\partial V/\partial Y}(X_0 - X, Y_0 - Y) \tag{19}$$

Suppose first that equation (19) fails to hold; we will conclude that (X, Y) cannot be Pareto-optimal. We can assume that

$$\frac{\partial U/\partial X}{\partial U/\partial Y}(X, Y) > \frac{\partial V/\partial X}{\partial V/\partial Y}(X_0 - X, Y_0 - Y)$$

Let b and j be small positive numbers such that

$$\frac{\partial U/\partial X}{\partial U/\partial Y}(X, Y) > \frac{j}{h} > \frac{\partial V/\partial X}{\partial V/\partial Y}(X_0 - X, Y_0 - Y)$$

and consider the allocation $(X + h, Y - j)$. We have

$$U(X + h, Y - j) \approx U(X, Y) + h \cdot \frac{\partial U}{\partial X} - j \cdot \frac{\partial U}{\partial Y} > U(X, Y)$$

$$V(X_0 - X - h, Y_0 - Y + j) \approx V(X_0 - X, Y_0 - Y) - h \cdot \frac{\partial V}{\partial X} + j \cdot \frac{\partial V}{\partial Y} > V(X_0 - X, Y_0 - Y)$$

contradicting Pareto-optimality.

On the other hand, if equation (19) *does* hold, then it is possible to show that (X, Y) must in fact be Pareto-optimal. Indeed, running the preceding argument backward shows that no allocation of the form $(X + h, Y - j)$ can be Pareto-preferred to (X, Y) when h and j are small. To show the same thing when h and j are arbitrary requires a little work using the convexity of indifference curves. If you are ambitious, you might try to complete the proof.

In competitive equilibrium, both Aline and Bob choose baskets where their marginal rates of substitution between X and Y are equal to the relative price of X in terms of Y. Because they both face the same relative price, it follows that their marginal rates of substitution are equal. But this is precisely the condition of equation (19). We conclude that a competitive equilibrium is Pareto-optimal. This is the theorem of the invisible hand.

Exercise

1. Aline's indifference curves are given by the family of equations $X^{1/2} \cdot Y^{1/2} = C$ and Bob's by the family of equations $X^{1/4} \cdot Y^{3/4} = C$. Aline owns 2 Xs and 5 Ys, while Bob owns 8 Xs and 5 Ys. Characterize the Pareto-optimal outcomes (i.e., give the equation of the contract curve) and compute the competitive equilibrium.

Answer: The equation of the contract curve is $3Y(10 - X) = X(10 - Y)$. In competitive equilibrium, Aline has $^{17}\!/_{3}$ Xs and $^{85}\!/_{28}$ Ys.

* Other references use slightly different formulations involving ≥ signs as well as > signs, but if U and V are continuous, the formulations are equivalent.

Chapter 10

1. Monopoly Pricing. The monopolist, like any producer, has a total revenue function $TR(Q) = Q \cdot P(Q)$, where $P(Q)$ is the maximum price at which demanders will purchase Q items. That is, $P = D^{-1}$, where D is the demand curve for the product. Differentiating, we find that the marginal revenue function is

$$MR(Q) = P(Q) + Q \cdot P'(Q)$$

Because $P'(Q)$ is negative, we conclude that for a monopolist, marginal revenue is always less than the price at which he sells his goods.

Note that $Q \cdot P'(Q) = P \cdot (1/|\eta|)$, where η is the elasticity of the demand curve. Thus, we can write

$$MR = P \cdot \left(1 - \frac{1}{|\eta|}\right) \tag{20}$$

To maximize profits, the monopolist (like any producer) chooses the quantity at which $MC = MR$. Since $MR < P$, it follows that for a profit-maximizing monopolist, $MC < P$.

2. Price Discrimination. Consider a monopolist who sells in two markets. In market A, the inverse to the demand function is $P_A(Q)$ and in market B, the inverse to the demand function is $P_B(Q)$. By selling Q_A items in market A and Q_B items in market B, the monopolist earns a total profit of

$$Q_A \cdot P_A(Q_A) + Q_B \cdot P_B(Q_B) - TC(Q_A + Q_B)$$

By differentiating separately with respect to Q_A and Q_B, we find that the conditions for profit maximization are

$$MR_A(Q_A) = MC(Q_A + Q_B) = MR_B(Q_B)$$

where MR_A and MR_B are the marginal revenue functions in the two markets. Combining this observation with equation (20), we discover that

$$\frac{P_A}{P_B} = \frac{\left(1 - \frac{1}{|\eta_A|}\right)}{\left(1 - \frac{1}{|\eta_B|}\right)}$$

where η_A and η_B are the elasticities of demand in the two markets.

Chapter 11

1. Collusion. Suppose that there are N firms in an industry, and that the ith firm has marginal cost curve MC_i. The inverse demand curve for the industry's product is given by the function $P(Q)$. Under competition, firms take the market price as given, so they produce quantities Q_i such that

$$MC_i(Q_i) = P\left(\sum_{i=1}^{N} Q_i\right)$$

This system of N equations in N unknowns determines the quantities Q_t.

Suppose alternatively that the firms collude in order to maximize industry profits. That is, the cartel seeks to maximize

$$\left(\sum_{j=1}^{N} Q_j\right) \cdot P\left(\sum_{j=1}^{N} Q_j\right) - \sum_{j=1}^{N} TC_i(Q_j)$$

The condition for this is that for each i,

$$MC_i(Q_i) = P\left(\sum_{j=1}^{N} Q_j\right) + \left(\sum_{j=1}^{N} Q_j\right) \cdot P'\left(\sum_{j=1}^{N} Q_j\right)$$

Note that the expression on the right is the industry's marginal revenue curve.

2. Cournot Oligopoly. Suppose that the N firms in an industry are not able to collude. Then each maximizes its profits subject to the constraints placed upon it by the behavior of other firms. However, this formulation is imprecise and ambiguous. Exactly what aspects of other firms' behavior shall we assume that each firm takes as given? In the *Cournot model* of oligopoly, the assumption is that each firm takes its rivals' *quantities* as given. Thus, the t^{th} firm attempts to maximize

$$Q_i \cdot P\left(Q_i + \sum_{ji} Q_j\right) - TC_i(Q_i)$$

treating each $Q_j (j \neq i)$ as a constant. This leads the firm to set

$$MC_1(Q_i) = P\left(Q_i + \sum_{ij} Q_j\right) + Q_i P'\left(Q_i + \sum_{ij} Q_j\right)$$

These N equations in N unknowns determine the quantities Q_i.

Chapter 15

1. The Derived Demand for Factors of Production. In the short run, the firm's demand curve for a factor is the inverse function to that factor's marginal revenue product, as discussed in the text. To derive the demand for labor in the long run, we assume that the firm has the production function $f(L, K)$, and we take as given the price of capital, P_K, and the price of output, P.

At any given wage rate P_L, the firm chooses quantities L of labor and K of capital to maximize its profit

$$P \cdot f(L, K) - P_L \cdot L - P_K \cdot K$$

The first-order conditions for a maximum are

$$P \cdot \frac{\partial f}{\partial L}(L, K) = P_L$$

$$P \cdot \frac{\partial f}{\partial K}(L, K) = P_K$$

These two equations in the two unknowns L and K determine the firm's employment of labor and of capital. If (L, K) is a solution to the system, then the quantity L corresponds to the price PL on the firm's long-run demand curve for labor.

Continuing to hold P_K and P fixed, let $L_0(P_L)$ and $K_0(P_L)$ be the profit-maximizing quantities of labor and capital when the wage rate of labor is P_L. Thus, the functions L_0 and K_0 are implicitly defined by the system

$$P \cdot \frac{\partial f}{\partial L}(L_0(P_L), K_0(P_L)) = P_L$$

$$P \cdot \frac{\partial f}{\partial K}(L_0(P_L), K_0(P_L)) = P_K$$

Differentiating with respect to the variable P_L, we get

$$P \cdot \frac{\partial^2 f}{\partial L^2}(\mathrm{L}_0(P_L), K_0(P_L)) \cdot \frac{dI_0}{dP_L}(P_L) + \frac{\partial^2 f}{\partial L \partial K}(L_0(P_L), K_0(P_L)) \cdot \frac{dK_0}{dP_L}(P_L) = 1$$

$$P \cdot \frac{\partial^2 f}{\partial L \partial K}(L_0(P_L), K_0(P_L)) \cdot \frac{dL_0}{dP_L}(P_L) + \frac{\partial^2 f}{\partial^2 K^2}(L_0(P_L), K_0(P_L)) \cdot \frac{dK_0}{dP_L}(P_L) = 0$$

Solving this system, we find that

$$\frac{dL_0}{dP_L}(P_L) = \frac{\partial^2 f/\partial K^2}{P \cdot \delta} \tag{21}$$

$$\frac{dK_0}{dP_L}(P_L) = \frac{-\partial^2 f/\partial L \partial K}{P \cdot \delta} \tag{22}$$

where

$$\delta = \left(\frac{\partial^2 f}{\partial K^2} \cdot \frac{\partial^2 f}{\partial L^2} - \left(\frac{\partial^2 f}{\partial L \partial K} \right)^2 \right)(L_0(P_L), K_0(P_L))$$

Because we assume that f satisfies the analogues of equations (1) through (5), we know that $\partial^2 f/\partial K^2 < 0$ and that $\delta > 0$. It follows from this and equation (21) that dL_0/dP_L is everywhere negative. That is, the firm's demand curve for a factor of production must be everywhere downward sloping. This is in contrast to the consumer's demand curve for a consumption good, where the Giffen phenomenon is at least a theoretical possibility.

2. Changes in the Price of Another Factor. In the preceding section we held the price of capital fixed and determined how the firm's employment of labor and of capital varied in response to a change in the wage rate of labor. In particular, we derived the equation for the firm's labor demand curve and showed that this curve must slope downward.

Equation (22) shows how the firm's employment of capital changes in response to a change in the wage rate of labor. Because δ is known to be positive, the sign of dK_0/dP_L depends only on the sign of the cross partial derivative $\partial^2 f/\partial L \partial K$. When the cross partial is positive, we say that capital and labor are *complements in production*, and when the cross partial is negative, we say that capital and labor are *substitutes in production.*

When labor and capital are complements in production, equation (22) shows that an increase in the wage rate of labor leads to a fall in the demand for capital (and similarly, an increase in the rental rate for capital leads to a fall in the demand for labor). When labor and capital are substitutes in production, the reverse is true.

Economists believe that labor and capital are more often complements than substitutes in production. For example, if labor and capital are the only two inputs and if the production function exhibits constant returns to scale, then we can show that labor and capital must be complements in production. To see this, write

$$f = \frac{\partial f}{\partial L} \cdot L + \frac{\partial f}{\partial K} \cdot K$$

and differentiate with respect to L to get

$$\frac{\partial^2 f}{\partial L^2} \cdot L + \frac{\partial^2 f}{\partial K \partial L} \cdot K = 0$$

Because $\partial^2 f/\partial L^2$ is negative, the cross partial must be positive as needed.

3. Changes in the Price of Output. Holding P_L and P_K fixed, we let the price P of output vary and write $L_0(P)$ and $K_0(P)$ for the profit-maximizing levels of labor and capital employment. Beginning with the system

$$P \cdot \frac{\partial f}{\partial L}(L_0(P), K_0(P)) = P_L$$

$$P \cdot \frac{\partial f}{\partial K}(L_0(P), K_0(P)) = P_K$$

we differentiate with respect to P and find

$$\frac{\partial^2 f}{\partial L^2} \cdot \frac{dL_0}{dP} + \frac{\partial^2 f}{\partial L \partial K} \cdot \frac{dK_0}{dP} = -\frac{\partial f}{\partial L}$$

$$\frac{\partial^2 f}{\partial L \partial K} \cdot \frac{dL_0}{dP} + \frac{\partial^2 f}{\partial K^2} \cdot \frac{dK_0}{dP} = -\frac{\partial f}{\partial K}$$

Solving for dL_0/dP and dK_0/dP, we find that

$$\frac{dL_0}{dP} = \frac{1}{\delta} \cdot \left(-\frac{\partial^2 f}{\partial L^2} \cdot \frac{\partial f}{\partial L} + \frac{\partial^2 f}{\partial L \partial K} \cdot \frac{\partial f}{\partial K} \right)$$

$$\frac{dK_0}{dP} = \frac{1}{\delta} \cdot \left(-\frac{\partial^2 f}{\partial K^2} \cdot \frac{\partial f}{\partial K} + \frac{\partial^2 f}{\partial L \partial K} \cdot \frac{\partial f}{\partial L} \right)$$

This shows that when labor and capital are complements in production, an increase in the price of output leads to an increase in the demand for both labor and capital.

4. The Distribution of Income. In equilibrium, the wage rate of labor is equal to its marginal revenue product $P \cdot \partial f/\partial L$, and the wage rate of capital is equal to its marginal revenue product $P \cdot \partial f/\partial K$. Thus, when labor and capital are the only inputs, the firm's total costs are

$$P \cdot \frac{\partial f}{\partial L}(L, K) \cdot L + P \cdot \frac{\partial f}{\partial K}(L, K) \cdot K$$

The total revenue of the firm is the price of output multiplied by the quantity of output, or

$$P \cdot f(L, K)$$

Finally, the profits of the firm are given by the difference between revenue and cost, or

$$P \cdot f(L, K) - P \cdot \frac{\partial f}{\partial L}(L, K) \cdot L + P \cdot \frac{\partial f}{\partial K}(L, K) \cdot K$$

From this expression we see immediately that a competitive firm earns zero profits if and only if it produces at a point where there are constant returns to scale. If returns to scale are decreasing, then the firm earns positive profits, and if returns to scale are increasing, then the firm earns negative profits.

In the long run, one can argue that all firms experience constant returns to scale provided that the production function really includes every factor of production. This is because of the principle that "what a firm does once, it can do twice" discussed in Chapter 6 of the textbook. It follows that when all payments to all factors are considered, the competitive firm earns zero profits in the long run.

Exercise

1. A firm produces according to the production function $f(L, K) = L^{1/4} \cdot K^{1/4}$. Holding fixed the prices of output and of capital, derive the firm's short-run and long-run labor demand curves. How does the demand for labor vary with the price of capital? With the price of output?

Answer: Fixing the price of output at 1 and the price of capital at P_K, short-run labor demand is

$$\left[\frac{K}{256P_L^4}\right]^{1/3}$$

and long-run labor demand is

$$\frac{1}{16P_K^{1/2}P_L^{3/2}}$$

Chapter 16

1. The Supply of Labor. To model the supply of labor, we assume that the worker's indifference curves between consumption and labor are given by the family of equations

$$V(L, Y) = \text{constant}$$

where L is labor and Y is consumption. Given a wage rate P_L, the worker who works L hours earns $P_L \cdot L$ units of consumption; thus, he chooses L so as to maximize

$$V(L, C_0 + P_L \cdot L)$$

where C_0 is 1the worker's nonlabor income.

The first-order condition for a maximum is

$$\frac{-V_1(L, C_0 + P_L \cdot L)}{V_2(L, C_0 + P_L \cdot L)} = P_L$$

Thus, the labor supply function S is implicitly defined by the equation

$$\frac{-V_1(S(P_L), C_0 + P_L \cdot S(P_L))}{V_2(S(P_L), C_0 + P_L \cdot S(P_L))} = P_L \tag{23}$$

Let L_0 be the total time available to the worker. For example, if we are deriving the supply of labor per day, then $L_0 = 24$ hours. Set $U(X, Y) = V(L_0 - L, Y)$ so that X can be thought of as leisure. We assume that U satisfies properties (1) through (5).

This guarantees that the first-order condition really is sufficient for the existence of a maximum.

The wage rate P_L can be viewed as the price of leisure, and the effect of a change in the wage rate can be decomposed into income and substitution effects, working with the function U just as in Chapter 4. Note that when leisure is a normal good, the income effect leads the worker to consume more leisure, which is the same thing as supplying *less* labor.

2. The Representative Agent. Suppose that there is a fixed amount of capital in society and that labor L produces output Y according to the total product function

$$Y = TP(L)$$

In an economy consisting of a single individual, that individual would choose the quantity of his labor input by maximizing the function

$$V(L, TP(L))$$

The first-order condition is

$$\frac{-V_1(L, TP(L))}{V_2(L, TP(L))} = TP'(L) \tag{24}$$

Given a wage P_1, each worker supplies a quantity of labor determined by equation (23). Each employer demands a quantity $D(P_1)$ of labor determined by the condition

$$TP'(D(P_L)) = P_L$$

For the representative agent, the quantity of labor demanded must coincide with the quantity supplied. Call this common quantity L_0. Then the representative agent employs L_0 units of labor, produces $Y_0 = TP(L_0)$ units of output, pays a wage bill of $P_t \cdot L_0$, and earns a nonlabor income C_0 equal to the difference between what he produces and his wage bill; that is

$$C_0 = TP(L_0) - P_L \cdot L_0 \tag{25}$$

In equilibrium, L_0 is also the quantity of labor supplied by the representative agent; that is, $S(P_L) = L_0$. Combining this with equation (23) and (25) and comparing with equation (24), we find that in a competitive economy, the representative agent supplies exactly the same amount of labor that he would choose to supply if he lived in isolation.

Chapter 17

1. A Two-Period Model. The simplest way to model the allocation of goods over time is to imagine an individual who lives for two periods. We then treat consumption in period one and consumption in period two as different goods and apply all of the consumer theory that we have developed.

Let C_0 and C_1 denote "consumption today" and "consumption tomorrow." Then the consumer's indifference curves are given by the family of equations

$$U(C_0, C_1) = constant$$

for some function U satisfying properties (1) through (5). We often assume that U is of the special form

$$U(C_0, C_1) = V(C_0) + \beta \cdot V(C_1)$$

where β is a constant satisfying $0 < \beta < 1$ and V is a function of one variable satisfying

$$V'(C) > 0$$

$$V''(C) < 0$$

Suppose that the consumer is endowed with E_0 units of consumption today and E_1 units of consumption tomorrow. Then if the price of consumption today in terms of consumption tomorrow is $1/(1 + r)$, the present value of the consumer's wealth is

$$E = E_0 + \frac{1}{1 + r} \cdot E_1$$

and his goal is to maximize $V(C_0) + \beta \cdot V(C_1)$ subject to the constraint

$$C_0 + \frac{1}{1 + r} \cdot C_1 = E$$

The first-order condition is

$$\frac{V'(C_0)}{V'(C_1)} = \beta \cdot (1 + r)$$

The representative agent must consume his endowment, so for him we have $C_0 = E_0$ and $C_1 = E_1$. If in addition $E_0 = E_1$, then it follows that in equilibrium we must have

$$\beta = \frac{1}{1 + r}$$

Answers to All the Exercises

Chapter 1

1.1 Demand for coffee rises. It depends on the related good.

1.2 It would probably rise.

1.3 The demand curve would shift downward a vertical distance 5¢. The demand curve would shift upward a vertical distance 10¢.

1.4 The demand curve would shift downward, but not parallel to itself, because the amount of the tax per item varies with the quantity purchased.

1.5 It would fall. It would fall because an increase in the price of leather belts would probably lead to an increase in the price of leather.

1.6 At a price of 40¢ per cup, suppliers get to keep 30¢ per cup and so supply 300 cups (read off the second line of Table A). And so forth.

1.7 Panel A. Price rises and quantity rises.

1.8 The sales tax causes very little change in price if either the demand curve is quite steep or the supply curve is quite flat. Price drops by nearly the whole 5¢ if either the demand curve is quite flat or the supply curve is quite steep.

1.9 Because the vertical distance from S to S' is 5¢ and the vertical distance from E to H is less than this.

1.10 You should shift the supply curve up a vertical distance 2¢ and the demand curve down a vertical distance 3¢. The new price to suppliers is 2¢ less than the new market price and the new price to demanders is 3¢ more than the new market price. If you have drawn your picture correctly, you will find that the new price to suppliers is on the old supply curve, the new price to demanders is on the old demand curve, and the distance between the two is 5¢. Because this can happen at only one place, the effect must be the same as that of the pure 5¢ sales and excise taxes.

Chapter 2

2.1 The relative price of bread is the reciprocal of the relative price of wine. When a number increases, its reciprocal decreases.

Examples include VCD-player, CD-ROM player, mobile telephones, beepers, and video cameras. Garten, Jeffrey E., Opening the Doors for Business in China

2.2 For the carpenter to rewire takes 20 hours, during which time he could perform 20/18 = 10/9 paneling jobs.

Chapter 3

3.1 B: 4 eggs, 7 root beers. C: 1 egg, 2 root beers. D: 4 eggs, 2 root beers.

3.2 Basket A, which has more of everything.

3.3 When you give Jeremy an egg, your stock of eggs is reduced from 7 to 6; when he gives you 4 root beers in exchange, your stock of root beers is increased from 2 to 6.

3.4 Imagine sacrificing 1 root beer in exchange for some eggs in such a way that the trade leaves you just as happy as you started out. This will bring you to a point on the indifference curve with vertical coordinate 1. The corresponding horizontal coordinate shows how many eggs you have at the end of the exchange; the excess of this quantity over the 7 eggs you started with shows the marginal value (to you) of a root beer.

3.5 If Jack sacrifices 1 egg for 6 root beers, he moves from point *C* to point *D*, staying on the same indifference curve. If Jill sacrifices 1 egg for 1 root beer, she moves from point *C* to point *E*, staying on the same indifference curve.

3.6 The consumer values additional root beers highly relative to additional eggs when he has few root beers and lots of eggs. Therefore, the indifference curve should be shallower toward the southeast, confirming our belief that indifference curves are convex.

3.7 Because it is on a higher indifference curve. The budget line would have to be flatter, intersecting the *x*-axis on a higher indifference curve than where it intersects the *y*-axis.

Chapter 4

4.1 The new budget line is shifted southwest from the original line, and parallel to the original.

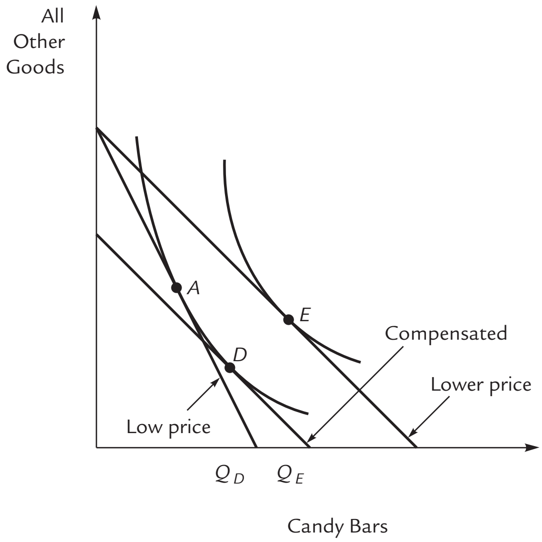
4.2 No. No. For *Y* to be inferior, point *B* would have to be located vertically below point *A*.

4.3 With an income of $12, the consumer chooses point *C* in panel *A*, and therefore consumes 12 eggs.

4.5 The budget line pivots inward around its x-intercept. If *Y* is not Giffen, the new optimum is vertically below the original. If *Y* is Giffen, the new optimum is vertically above the original.

4.6 When the price of *X* is \$6, the consumer chooses point *C* in Exhibit 4.8A and therefore consumes 2 eggs.

4.7



The movement from *A* to *D* represents the substitution effect, and the movement from *D* to *E* represents the income effect.

4.8 It means that when income rises, quantity of *X* falls; in other words, *X* is an inferior good.

4.9 For salt, a 10% price increase is associated with a 1% quantity decrease, so price elasticity = –1%/10% = .1. For tomatoes, price elasticity = –46%/10% = 4.6.

4.10 4.1%. 7.3%. 1.4%.

Chapter 5

5.1 The numbers decrease in this case because the farmer sprays the most productive acres first and less productive acres later. The total benefit of spraying 3 acres is the sum of the marginal benefits on the first, second, and third acres.

5.2 The farmer still sprays 4 acres, because the marginal benefit and marginal cost columns remain unchanged.

5.3 Yes. Now the marginal cost numbers are all \$1/acre instead of \$3/acre. The farmer now sprays 6 acres.

5.4 \$8 per dress. \$3 per dress.

Chapter 6

6.5 In the first row, \$3 per dress = \$15 per worker/5 units per worker, and so forth.

6.7 For given quantities *L* and *K* of labor and capital, the isoquant through (*L*, *K*) shows the maximum quantity of output that can be produced with this basket of inputs. Because there is only one such maximum quantity, there can be only one isoquant through a given point. Because there is always some quantity that can be produced with (*L*, *K*), there is always an isoquant through any given point.

6.8 At *E*, the isocost is steeper than the isoquant, so $MRTS_{LK} < P_t/P_K$. If the firm hires one less unit of labor and $MRTS_{LK}$ additional units of capital, it can stay on the isoquant, decrease its labor costs by P_L, and increase its capital costs by only $MRTS_{LK} \cdot P_K$, which is less than the decrease in labor costs. Total costs are decreased by this move, so it is a wise one for the firm. Having moved to the northwest, the firm continues moving in this direction until it reaches the point of tangency, *C*.

6.9 \$145. \$60. \$77.50.

6.10 A 1% increase in output requires more than a 1% increase in all inputs. Therefore, average cost increases when output increases.

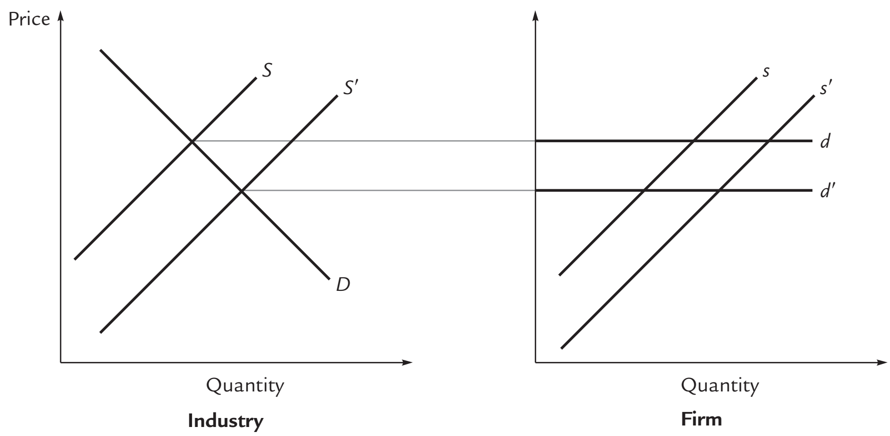
6.11 \$115. \$137.50. \$165.

6.12 The medium plant is best; the large plant is second best. In the long run the firm chooses the medium plant. At Q_2, the $SRAC_2$ curve is tangent to the *LRAC* curve.

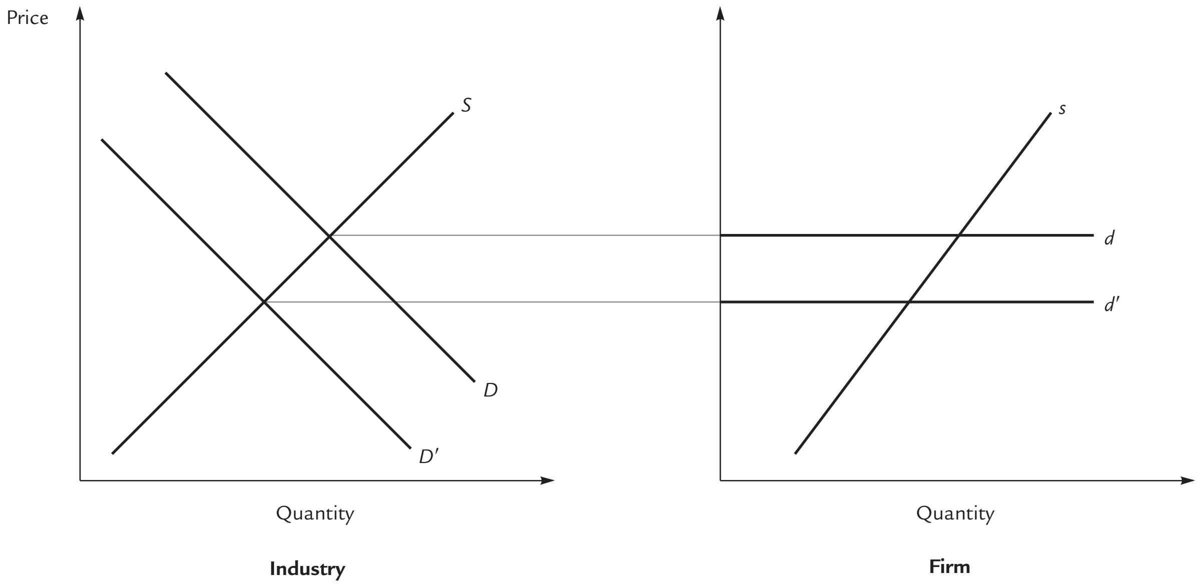
Chapter 7

7.1 Firm A produces 4, firm B produces 6, firm C produces 7, and the industry produces 17.

7.2



7.3



7.4 $11.

7.5 The demand curve rises, and all effects are opposite to those shown in Exhibit 7.22.

Chapter 8

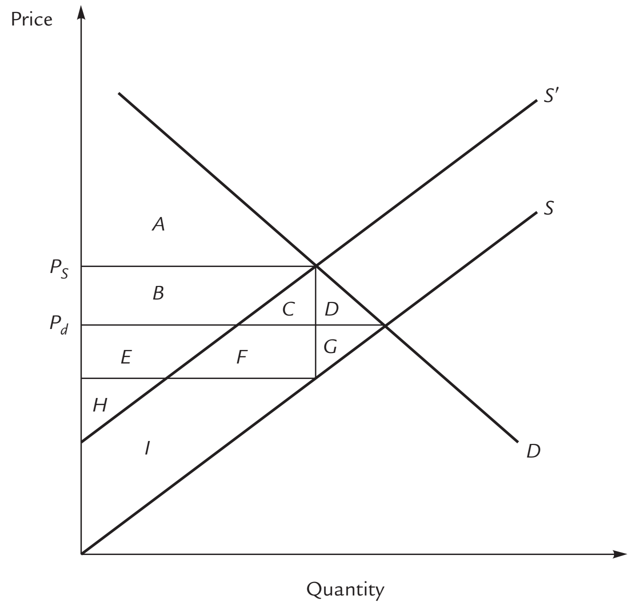
8.1 The entries are $8, $14, $17, $17, $15, $10. The largest of these, 17, occurs at a quantity of 4.

8.2 5.

8.3 Consumers lose $C + D + E$. Producers lose $F + G + H$. Tax recipients gain $C + D + F + G$. Yes.

8.4 Refer to the graph at the top of the next page.

	Before Taxation	With Excise Tax
Consumer Surplus	$A + B + C + D$	A
Producer Surplus	$E + F + G + H + I$	$H + I$
Tax Revenue	—	$B + C + E + F$
Social Gain	$A + B + C + D + E + F + H + I$	$A + B + C + D + E + F + H + I$
Deadweight Loss	$+ G$	$D + G$

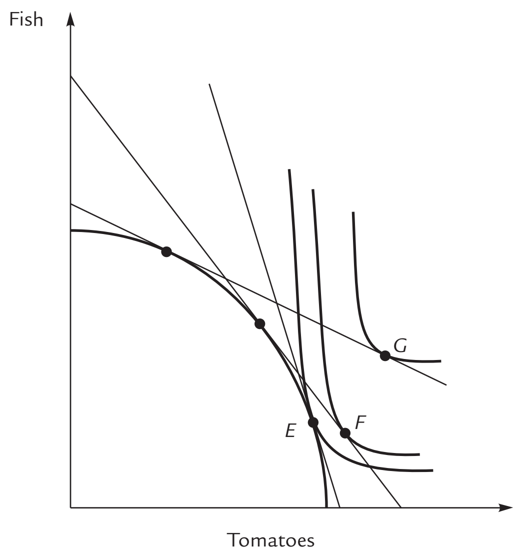


(Warning: these lettered areas are not the same as those in Exhibit 8.8.)

8.7 These calculations would be just like those of Exhibit 8.8. There are no gains or losses to the Japanese producers because their supply curve is flat.

8.8 In terms of Exhibit 8.17, take from the consumers $G + \frac{1}{2}H + I + \frac{1}{2}J$. Give the producers $G + \frac{1}{2}H$ and give the tax recipients $I + \frac{1}{2}J$. (This is only one of many possible solutions.)

8.9 In this drawing, the autarkic relative price is the slope of the budget line through *E*. The budget line through *F* results when the world price of tomatoes is slightly lower, and the budget line through *G* results when the world price is lower still. As the world price deviates more from the autarkic price, Robinson moves to higher indifference curves and becomes better off.



Chapter 9

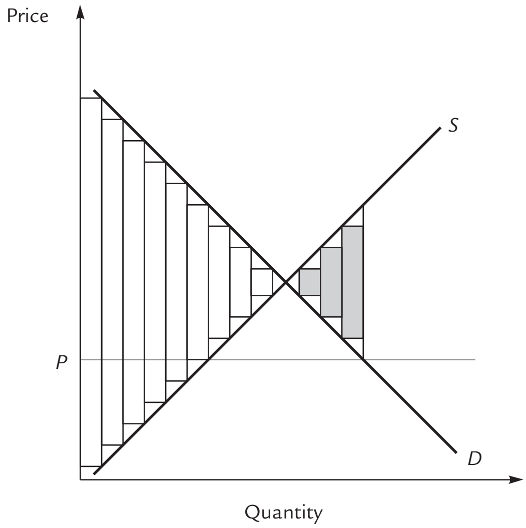
9.1 Total value = $54. Total cost = $35. Social gain = $19.

9.2 Total value = $46. Total cost = $35. Social gain = $11.

9.3 Give Curly's second egg to Moe. Simultaneously, take from Moe any amount of money between $3 and $11 and give it to Curly.

9.4 $8.

9.5 White rectangles are gains and shaded ones are losses.



9.7 Under a limited draft, $B + C$ represents the amount by which each soldier's wages are reduced from equilibrium, times the number of soldiers. Thus, it is wealth transferred from soldiers to consumers.

9.8 In terms of Exhibit 9.7, the confiscation of rents adds *A* to the pockets of those who confiscate and subtracts *A* from Jennifer's producer surplus (leaving him with zero).

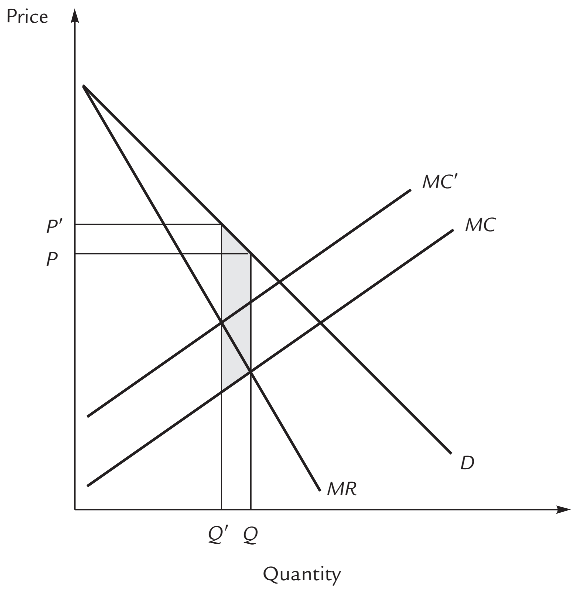
9.9 From $100 to $120. From $50 to $60.

9.10 The plumber today receives $100 to fix a leak. $100 will buy 20 movie tickets at $5 apiece. Overnight, all prices double, but the plumber thinks they have tripled. Tomorrow, he is offered $200 to fix the leak. Although $200 will still buy 20 movie tickets (now at $10 apiece), the plumber thinks that it will buy only 13.33 movie tickets (which he now believes sell at $15 apiece). Thus, he thinks he is being offered fewer movie tickets per plumbing repair than he is really being offered. This mistake leads him to supply less plumbing service.

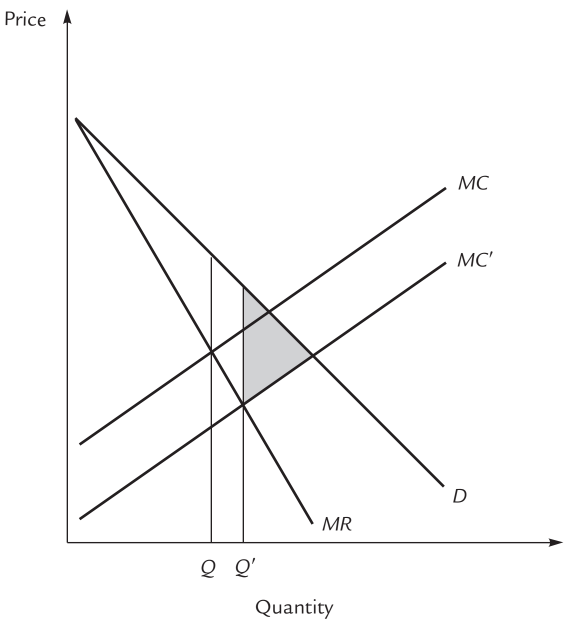
Chapter 10

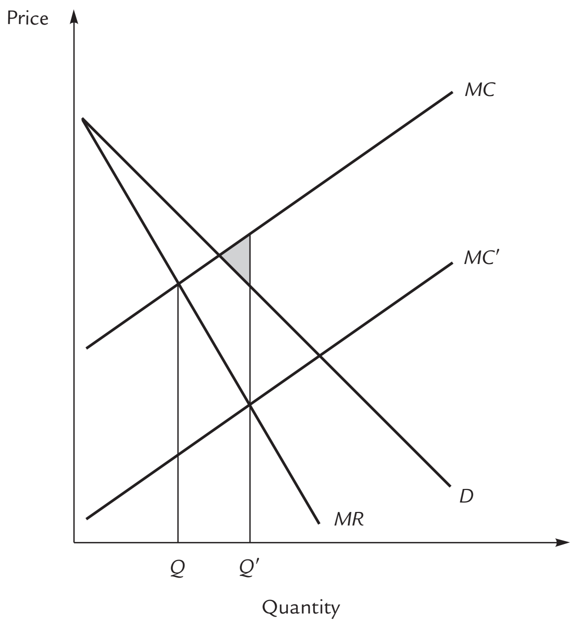
10.1 $7 − $3 = $4. Yes.

10.3 The shaded area is additional deadweight loss due to the excise tax.



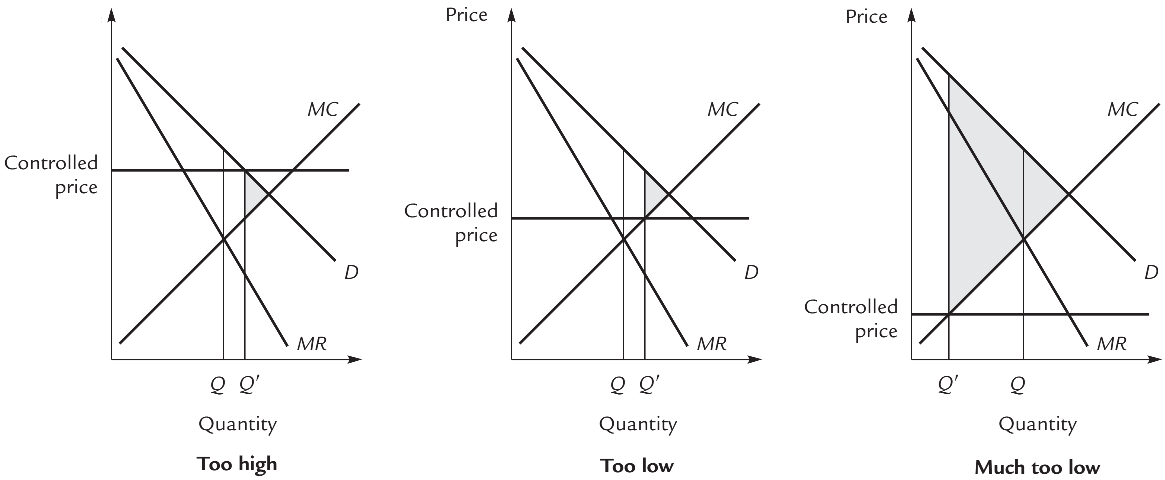
10.4 The shaded areas are deadweight loss.





10.5 There are no waiting lines because the monopolist's price and quantity are given by a point on the demand curve. Thus, the quantity demanded is equal to the quantity supplied.

10.6

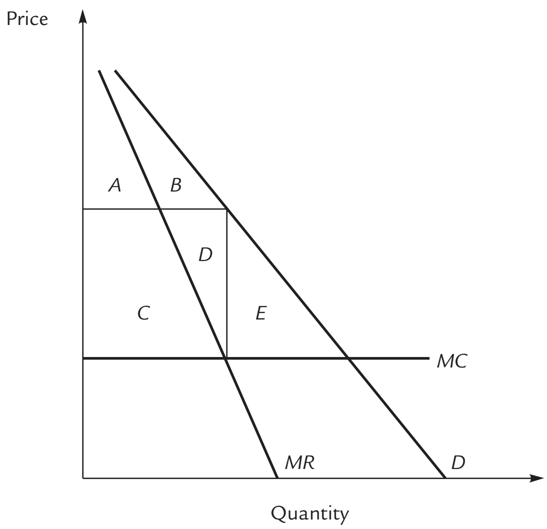


10.7 Lobbying uses up resources. Bribery merely transfers resources from one individual to another.

10.8 If marginal revenue in the adult market is greater than in the children's market, he can sell one more haircut to an adult and one less to a child, increasing his revenue without affecting his cost. Similarly if adults and children are reversed. Thus, Benjamin is never satisfied if the two *MR*s are different. He also wants $MR = MC$ just like any firm.

Chapter 11

11.1 Social welfare is unambiguously reduced by the merger, from $A + B + C + D + E$ to $A + B + C + D$.



11.2 An appropriate price is any amount greater than $A + B$ but less than $A + B + E + H$.

11.3 If they produce anything less in the way of services, then they earn positive profits, leading them to compete with each other for additional customers by increasing the service level.

11.4 To see that $V > P_1 - P_0$, examine the vertical line at Q_1 in Exhibit 11.4. The portion of this line that stretches from MC′ to D has length V, which is clearly greater than $P_1 - P_0$. To see that $A + B > A + C$, note that the two triangles are similar, so it suffices to check that the base of $A + B$ is longer than the base of $A + C$. That is, we must check that $Q_1 > Q_0$, which is given.

11.5 At price P_0, firms produce quantity Q_0. At this quantity, average cost is also P_0, so firms earn zero profits.

11.6 Because the other sellers in the marketplace provide close substitutes for the given seller's product.

11.7 The one closer to an endpoint would jump around the other one.

Chapter 12

12.1 10 calories, 15 calories.

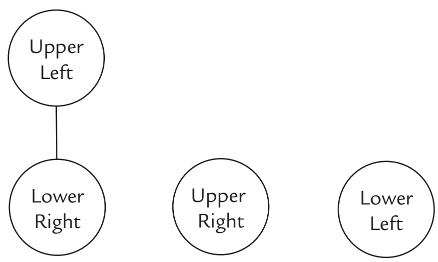
12.2 In the lower left, B wants to switch. In the lower right, both want to switch.

12.3 In the upper right, Ditto wants to switch. In the lower left, Ditto wants to switch. In the lower right, Dot wants to switch.

12.4 Because if you play this strategy, a wise opponent will always play "paper", beating you twice as often as you beat him. But if your opponent plays "paper" consistently, you'll want to play "scissors" consistently; that is, you'll deviate from the proposed strategy.

12.5 Because the weak pig prefers both C and D to B. Because the strong pig prefers both A and B to C.

12.6 The upper left, upper right, and lower left are Pareto optima.



12.7 Because any shift away would hurt B.

12.8 Because any shift away would hurt A.

12.9 In each case, a move to either of the Pareto optima benefits both Fred and Ethel.

12.10 In each case, at least one firm can do better by changing its strategy.

Chapter 13

13.1 The demand curve would lie entirely above the social marginal cost curve.

13.3 The White Sox. The White Sox.

13.4 New York. New York.

Chapter 14

14.1 The difference is \$5 − \$0 = \$5. The \$5 benefit to the sixth visitor is completely offset by the costs of \$1 apiece that he imposes on each of the first 5 visitors.

Chapter 15

15.1 As labor input increases, so does output, and therefore the output price falls. The effect is to steepen the MRP_t curve.

Chapter 16

16.1 As you move up and to the right you trade away leisure for consumption, so the marginal value of leisure increases.

16.2 If the wage were greater than the marginal value of leisure, the worker could gain by working more. If the wage were less than the marginal value of leisure, he could gain by working less.

16.3 If occupation A were more attractive than occupation B, then workers in occupation B would switch over to occupation A, raising wages in occupation B and lowering them in occupation A. This would continue until the two occupations were equally attractive.

Chapter 17

17.1 2 apples tomorrow per apple today. 100%.

17.2 The price is 4 apples. The face value is 5 apples. The discount is 1 apple.

17.3 .44 apples today. .30 apples today.

17.4 .76 apples today.

17.5 1.10 apples today.

17.6 \$20.

17.7 8%. 3%.

17.11 At an interest rate of 10%, Barb demands 5 units of current consumption.

17.12 Rebecca's budget line is steeper than in the exhibit and her optimum lies to the northwest of point *E*. She wants to be a net lender, consuming less than her current endowment. Therefore, people on average want to lend and this places downward pressure on the interest rate.

Chapter 18

18.1 Bet $100 on heads.

18.2 The expected value of basket C is always $100, regardless of whether the coin is biased. If the coin comes up heads 2/3 of the time, the expected value of basket D is $113.33. If the coin comes up tails 2/3 of the time, the expected value of basket D is $86.67.

18.3 For an unbiased coin, the iso-expected value lines have slope –1 and *C* and *D* lie on the same line. For a coin that comes up heads 2/3 of the time, the iso-expected value lines are steeper and *D* is on a higher line than *C*. For a coin that comes up tails 2/3 of the time, the lines are shallower and *C* is on a higher line than *D*.

18.4 It would be that portion of the black line in Exhibit 18.3 that lies to the right of point C.

18.5 5 to 1 ½ to 1.1 to 1.

18.6 $15 worth.

18.7 For the advertising campaign fair odds are 1/2 to 1. The actual odds are 1 to 1. The expected winnings are $500. For the concert, fair odds are 1 to 1 and actual odds are 3 to 1. The expected winnings are 50¢.

18.8 Owners sell more today and less in March, driving down the current price and driving up the March spot price until the two are equal.

18.10 Expected return is 50%; $\sigma = 0$. Thus, the point is to the left of *GSS* and on the vertical axis.

18.11 $5. $1.

18.12 The actual price is sometimes as high as $2, sometimes as low as $0, and $1 on the average day.

18.13 $4. $5. $6.

18.14 $4. 800 heads. $1. $3. $5.

18.15 Price of lettuce = (1/50) × (Lumberjacks' income) + 50¢.

Answers to Problem Sets

This appendix contains answers, hints, and discussions for many of the end-of-chapter problems throughout the book. In some cases, you will find complete answers with reasons. In others, you will find answers without reasons; it is still your job to provide the reasons. In still others, you will find hints but no answers. In a few cases, you will find complete answers together with additional related discussion that goes beyond what is necessary to answer the problem correctly.

Chapter 1

1. False, in the sense of "not necessarily true." This new device will lead to an increase in the quantity demanded of those activities that are capable of leading to unwanted pregnancies; therefore the number of unwanted pregnancies could either rise or fall.

Students sometimes object that there are people who won't use birth control no matter how safe, cheap, effective or easy it is. That's likely to be true, but irrelevant. Given the long list of advantages of this device, it's a save bet that *some* people will use it, which suffices to affect the number of pregnancies.

4. False. The demand curve for apartments shifts downward; therefore the price falls.

8. *Hint:* What happens to the demand curve for meat? What happens to the equilibrium quantity of meat supplied?

12a. 50¢, $2.50, 4 pounds.

14a. Supply shifts left, so price rises and quantity falls.

14d. As farm workers move to the city to earn the higher wages, the supply of corn falls. Price rises and quantity falls. Sometimes students argue that wealthier industrial workers will demand more corn and therefore the demand curve shifts out as well. This is a commendable insight, but it overlooks the fact that those higher wages are paid by employers, who might now *reduce* their demand for corn, offsetting the additional demand by the workers. Therefore, unless we know more about why wages went up, we need not expect the market demand curve to shift.

16. When the supply curve shifts up by the amount T of the tax, the new equilibrium point is exactly a distance T above the old equilibrium point. The market price rises by the full amount of the tax.

Students commonly reach the correct answer *true* while offering a reason that is quite mistaken. Their (incorrect) argument is this: A vertical demand curve indicates that demanders will pay any price at all for lettuce; therefore, suppliers are able to pass the tax on completely without losing any sales. The argument is incorrect because it overlooks the fact that suppliers compete with each other. Any given supplier will indeed lose sales if he fails to match the going market price.

Indeed, to see that the argument cannot possibly be correct, ask yourself why suppliers don't raise their prices *before* the tax increase. If suppliers charge $1 originally and $1.25 after the imposition of a 25¢ tax, why don't they charge $1.25 (or more) even *before* the tax is imposed? The reason is that price is determined not by individual suppliers, but by the intersection of supply and demand.

23a. Possibly true.

23b. Certainly false.

28. The price of a shower rises by more than $50 but less than $200, leaving both buyers and sellers worse off.

Chapter 2

2. You may conclude that he is confused. If the relative price of widgets in terms of gadgets has risen, then the relative price of gadgets in terms of widgets must have fallen.

9. False, in the sense of "not necessarily true." The statement of the problem omits the key information that Mary is a highly skilled neurosurgeon, whereas George can do nothing except type. Mary's greater typing speed does not imply that she has a comparative advantage at typing.

Some students argue that *if* you are an employer who only wants to hire a typist, and *if* George and Mary are available at the same wage rate, then yes, it makes more sense to hire Mary as a typist than to hire George. But even this strained interpretation does not lead to the alleged conclusion. If you can really hire Mary at typist's wages, then you should set her to performing brain surgery, collect her fees as revenue to your firm, and use a small part of that revenue to hire George to do the typing.

10. False. Suppose that the going wage for child labor on farms is $5 per hour. Then the farmer without children must pay $5 to employ someone else's children; the farmer *with* children must forgo $5 per hour (which he could earn by renting his children out to neighboring farmers) to employ his own children. Both face the same cost of $5 per hour.

Some students argue that the farmer with children incurs the costs of feeding, housing, and education. However, it is *not* correct to count these among the costs of putting the children to work, because they must be paid whether the children work or not.

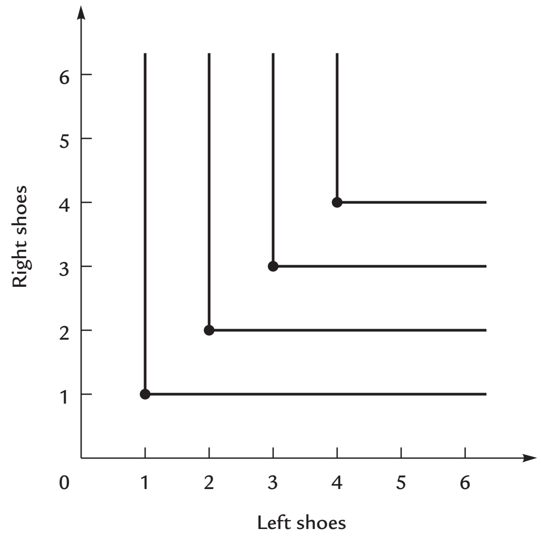
Other students argue that the farmer with children is wealthier at the end of the year because he makes no cash payments to hire labor. Whether or not this is true, it is irrelevant to the question. The question does not ask which farmer is wealthier, it asks only which farmer has higher costs of harvesting. The answer is that both have the same costs.

15. It goes wrong exactly where it says that in each case there are the same costs for producing, shipping, and marketing the clothes. If a professional middleman can perform some of these tasks more cheaply than Anderson-Little can, then Brand X might be able to pay the middleman more than enough to cover his costs and still deliver the clothes more cheaply than Anderson-Little.

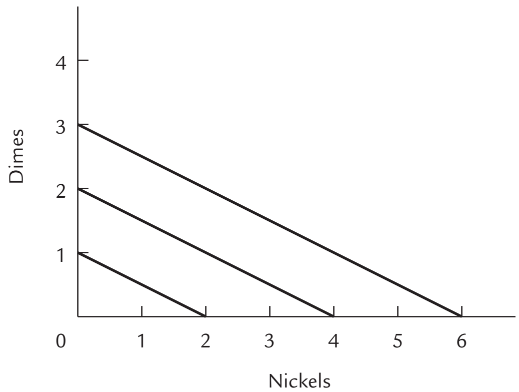
Chapter 3

1. False. A change in price is a change in opportunities, not a change in tastes.

2.



3.



7. Huey prefers both (1, 3) and (2, 2) to (3, 1).

9. In 2011, you are happier, eat more pizza, and drink less beer.

12. It's better to lose the $6.

16. You should be able to draw a single family of indifference curves consistent with both Amelia's and Bernard's choices. It is possible that Amelia and Bernard share this family of indifference curves; in other words, it is possible (though not certain) that they have identical tastes.

Assuming that Amelia and Bernard *do* have identical tastes, can you determine which of them is happier?

19. Of course not. The information given concerns opportunities, not tastes.

21. The program makes him worse off.

23d. It was a mistake for Pullman to institute the tax.

Chapter 4

1a. Fewer.

b. No.

5a. C, D, E.

b. B.

c. C, D.

8e. True.

8f. True.

12. Probably up.

13. False; in fact, shoes *must* be inferior.

15b. True.

19. She is happier in May and eggs must be inferior.

25. Income elasticity is 1; price elasticity is –1.

26. It means that when your income goes up, your consumption of the luxury good increases by more than your income does. If your income increases by 1%, your consumption of luxury goods increases by more than 1%. But you cannot increase your consumption of *all* goods by more than 1% without violating the budget constraint. Therefore, not all goods can be luxuries.

In fact, this can be made more precise. When your income increases by 1%, your expenditures must increase by exactly 1% "on average" over all goods.

Thus, the average income elasticity over all goods must be 1. In the averaging process, goods must be weighted by the percent of your income that you spend on them. Suppose that you consume only X and Y. Write k_x for the fraction of your income that you spend on X, k_Y for the fraction of your income that you spend on Y_1, η_x for your income elasticity of demand for X, and η_Y for your income elasticity of demand for Y. Then we must have

$$k_X\eta_X + k_Y\eta_Y = 1$$

If you want to prove this formula, start with the expressions

$$k_X = P_X X / I$$

$$k_Y = P_Y Y / I$$

$$P_X DX + P_Y DY = DI$$

(First explain what each of these expressions means and why it is true.) Then insert the expressions for k_X, k_Y, η_X, and η_Y into the final formula and simplify.

30f. Somebody whose income is derived entirely from wages feels a greater income effect from a change in the wage rate. Because the income effect works opposite to the substitution effect, such a person will respond less to a wage change than will somebody who has a lot of nonlabor income. Therefore, the person whose income is entirely from wages can be expected to have the steeper labor supply curve.

Chapter 5

2. True.

3. (i) has no effect; (ii) does have an effect. Make sure you can explain why.

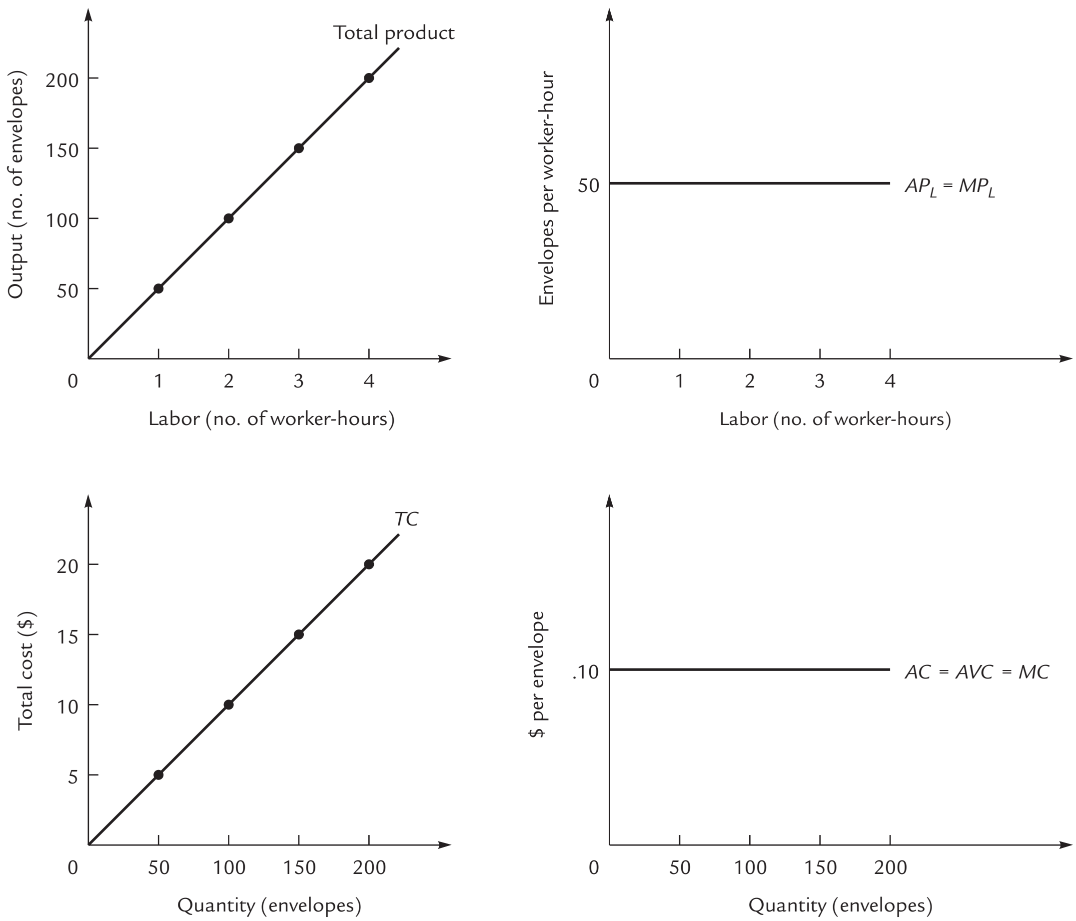
6. True. (Be sure you can explain why!)

11. The firm produces four items at $14 apiece.

13. If the area consists entirely of stores, then Wilma is correct. Rents are fixed costs that do not affect prices. The reason that rents are high is that stores are willing to pay a lot for this location, where prices are high. But if many of the buildings in the area are used for office space, or anything other than stores, then Fred might be right. The high rents (caused perhaps by a high demand for office space in this location) have driven some stores out of the area, raising the demand for the products of those that remain, and consequently increasing prices.

Chapter 6

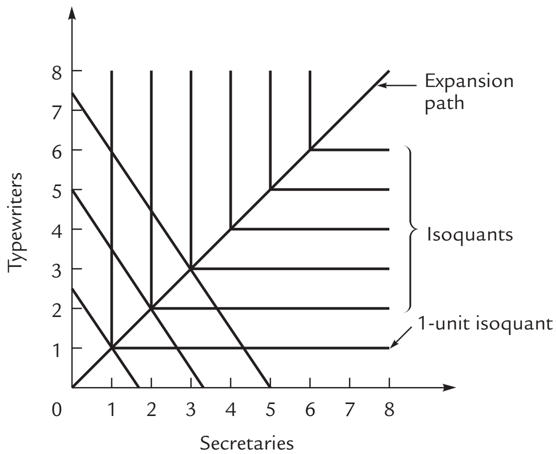
1.

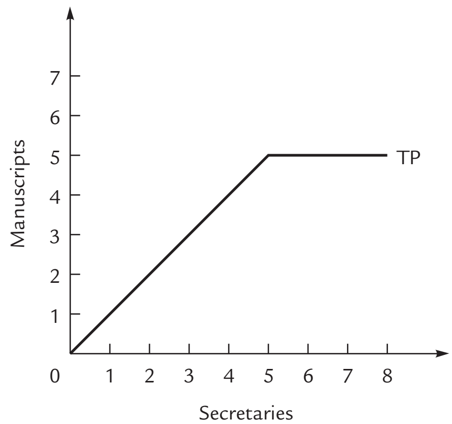


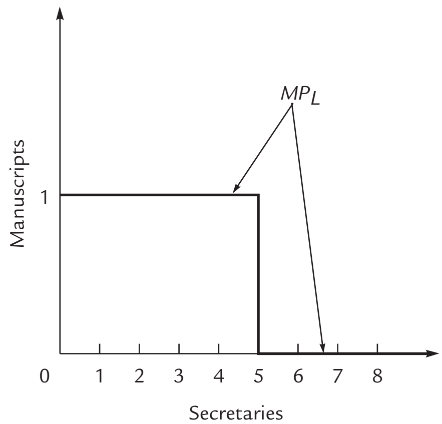
4. The first two rows look like this:

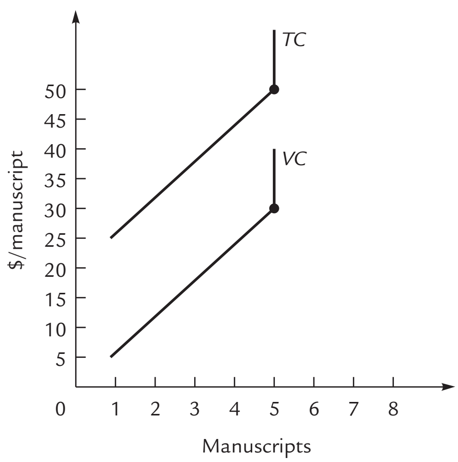
Quantity	VC	TC	AC	AVC
1	\$12	\$42	\$42	\$12
2	\$20	\$50	\$25	\$10

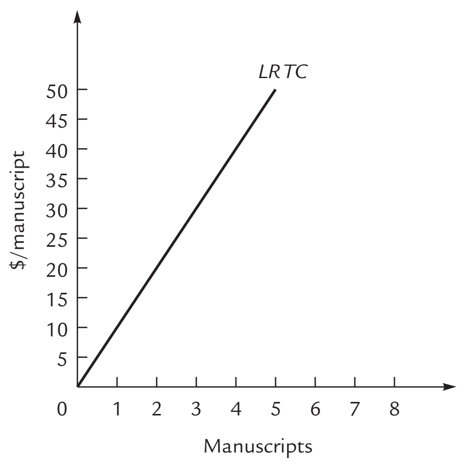
11.

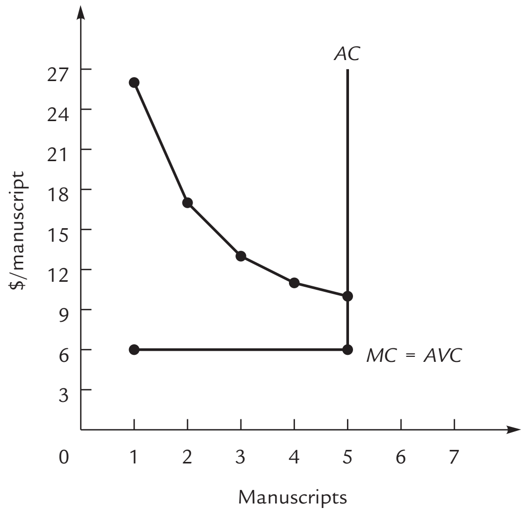


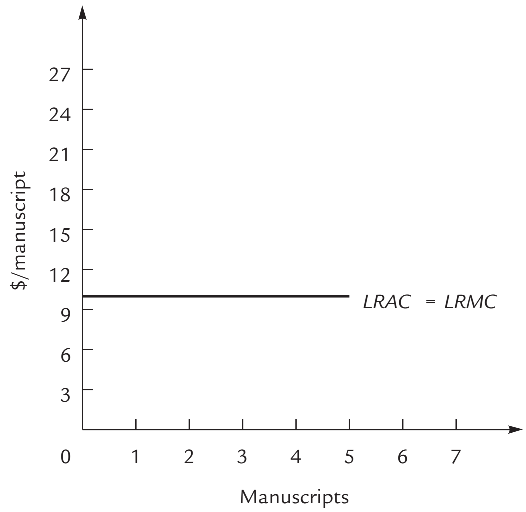












Chapter 7

1a. No change.

c. Price falls; quantity falls.

e. No effect.

g. Price rises; quantity rises.

i. No effect.

k. Price falls; quantity falls.

m. No change.

3. False.

10a. Price falls; quantity falls.

c. Price and quantity unchanged.

e. No effect.

g. No effect.

i. Price rises; quantity rises.

k. Gus leaves the industry.

m. Gus leaves the industry.

13a. Short Run. (There is no change at all in the long run.)

b. Long Run.

c. Short Run.

d. Short Run.

26. The marginal cost of providing a gallon of gasoline has risen by 50¢; thus the supply curves for both the firm and the industry shift vertically upward by 50¢ per gallon. In the constant-cost case, price rises by 50¢ per gallon; in the increasing-cost case it rises by less (in the long run).

Chapter 8

2a. From buying the widgets.

2b. A gadget.

3. By the Pareto criterion, (c) is better than (e) and no other comparisons can be made. By the efficiency criterion, (a), (b), (c), and (d) are all equally good, and all are better than (e).

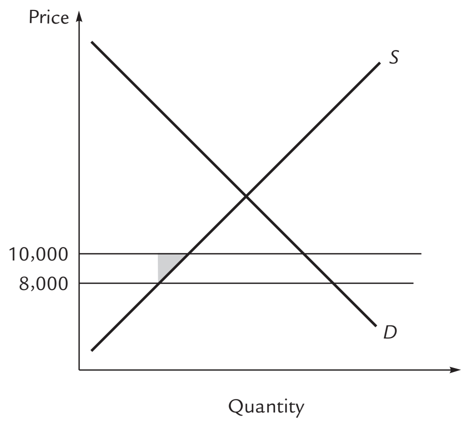
6. One solution is to give consumers $F + G + \frac{1}{3}E$ and give producers $C + D + \frac{1}{3}E$, taking the necessary resources from the taxpayers.

7. False. The line lengths adjust so that the value of waiting time is \$9 in both countries.

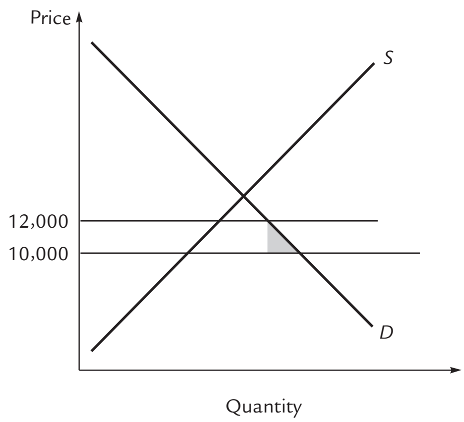
12. True. To see why, ask what happens to the "price to demanders" in the wheat market; then note that this price is part of the marginal cost of producing bread.

20. The triangle on the left reflects the fact that people purchase foreign cars that could have been produced more cheaply at home. The triangle on the right reflects the fact that people now buy fewer cars. Can you explain why?

21. The shaded area is deadweight loss:



23. The shaded area is deadweight loss:



34. True because the supply of robbers is horizontal.

Chapter 9

2. None of the council members has the expertise to determine the extent of the risk from the chemical plant; none has the expertise (or the information) to determine the extent of the benefits. It is reasonable to expect that the company owners and their insurers (who are experts at assessing risks) have access to more information than the council has. Under the councilman's proposal, they have the incentive to make use of that information. If the chemical company agrees to bear all of the costs of reimbursement, we may infer that it expects to earn enough from the plant to more than cover those costs. Similarly, if the insurance company is willing to bear the risk in exchange for a price that the chemical company is willing to pay, we may infer that it expects the amount of damage to the townspeople to be less than the gains to the chemical company.

The councilman's suggestion creates an incentive for those with easiest access to the relevant information to analyze that information and act on it in a socially desirable way.

5b. \$12.

10. Consumers could be made worse off if the pizzas are distributed to those who value them relatively little.

15. *Hint:* Suppose that all 3 of the applicants with incomes more than \$70,000 have revealed their incomes. Your income is exactly \$70,000. What will you do?

Chapter 10

2. False, because of the phrase "unlike competitors." Anybody can charge any price he wants to for anything. However, there is only one profit-maximizing price to charge, and it is foolish to deviate from this price, whether you are a monopolist, a competitor, or anything in between.

4. It drops a vertical distance \$1.

10. The social loss from monopoly is \$2.

13. True.

15. The firm sells 7 sweaters, 4 to men and 3 to women.

18. This appears to be price discrimination in favor of U.S. tourists, which would require that U.S. tourists have a greater elasticity of demand for meals at these restaurants than the natives do. But this would appear to be the exact opposite of the truth: Tourists, if they are to eat at all, must eat at restaurants, whereas natives have the option of eating at home. Also, tourists are less likely than natives are to know about alternative, out-of-the-way places to eat.

This suggests looking for an explanation that does not involve price discrimination. That is, we must ask why these restaurants find it less expensive to serve tourists than to serve natives. One wild guess is that tourists, for some reason, are better tippers, so that they actually pay more (inclusive of tip) for their meals than the natives pay. Of course, this keeps the staff happy and enables the management to pay lower wages; hence, serving tourists helps to keep their costs down.

We repeat that this explanation is a wild guess. If you have a better one, please send it to the author in care of South-Western.

32a. \$15.

b. At 60¢, profit (including admission fees) is \$35. At 50¢, profit is \$36. At 40¢, profit is \$34.

Chapter 11

2. Conceivably a vertical merger could be used to prevent resales. Suppose that a monopoly steel manufacturer wants to sell cheaply to automakers and expensively to construction firms. The steel firm worries that automakers can buy cheaply and resell to construction firms. But if the steel firm acquires an automaker as a subsidiary, it can sell cheaply to the automaker while ordering it not to engage in resales.

4. It would increase in the first case and decrease in the second.

9. One frequently cited alternative theory is that the manufacturer is acting as an enforcer for a cartel among the dealers. Under what circumstances do you find this theory either more or less plausible than the theory that is elaborated in the textbook?

12. False, because of the Prisoner's Dilemma. Each worker can rationally calculate that his own voluntary contribution is unlikely to be critical in determining the success of the union; therefore, he chooses not to join. It is important to notice that workers will elect not to join regardless of whether or not they believe that others are joining. It is possible that all workers could benefit from an outside enforcer who requires them to unionize.

15. The industry output is equal to $N/(N+1)$ times the output of a competitive industry. When N is large, the Cournot industry's output and the competitive industry's output are approximately equal.

16. The industry output is 3/4 of what it would be under competition, which is greater than what it would be under Cournot behavior. The first firm produces twice as much as the second firm and is better off than the second firm.

Chapter 12

1. I. (Right, Down).

1. III. (Right, Up).

1. V. None.

1. VII. (Left, Up) and (Right, Down).

2. I. (Right, Up), (Left, Down), and (Right, Down).

2. III. (Right, Up).

2. V. (Left, Down) and (Right, Down).

2. VII. (Left, Up) and (Right, Down).

3. I. Yes, Yes.

3. III. Yes, No.

3. V. No. No.

3. VII. No. No.

4. I. (Right, Down).

4. III. (Right, Up).

4. V. (Right, Down).

4. VII. (Right, Down).

Chapter 13

4. False. According to the Coase theorem, couples will negotiate so that housework is performed by the efficient provider.

In existing marriages, there might be an income effect whereby husbands, feeling power because of the new law, will choose less leisure and more housework. But in new marriages, there is no income effect because men can opt out of marriage entirely.

Therefore, existing marriages, maybe. In new marriages, false.

7. Assume first that there are no transactions costs between the beekeeper and the car dealer. In that case, your decision does not matter in the sense that it has no effect on the number of bees that are kept, the procedures used to contain the bees, the number of cars sold, the investment in tents by the car dealer, whether the car dealer will move away, and so forth. It matters in the sense that the beekeeper prefers one decision and the car dealer another.

Alternatively, if there *are* transactions costs, then all of the things that were left unaffected in the preceding paragraph can indeed be affected. A ruling for the car dealer could induce the beekeeper to rein in his bees (say with better netting) or to scale back his operation, while a ruling for the beekeeper could induce the car dealer to erect a tent or to move.

Because there are only two parties and they are in close proximity, the assumption of *no* transactions costs seems the more reasonable.

If a large collection of motorists is involved, the transactions costs can become considerable. It is difficult for the motorists to collectively negotiate with the beekeeper, particularly if different motorists are affected on different days. Many motorists might not even recognize the source of the problem.

In this case, some factors relevant to the decision are: How much would it cost the beekeeper to prevent his bees from flying over the roadway, either by containing them or moving elsewhere? What alternatives are available to motorists? Can they easily take a different route or would it be very costly to do so? How much damage do the bees actually do to the cars, and how much does it cost motorists to cope with this damage, either by having it repaired or by deciding to tolerate it?

22g. The Optimal tax is *N*+*O*+*P*.

25e. Expectation damages induce Betty to behave efficiently.

26. It does *not* follow that expectation damages are the appropriate standard. Although expectation damages lead to efficient breaches of contract, they might not lead to an efficient number of contracts being signed in the first place. A full analysis of the problem must account for the fact that the number of contracts signed will vary depending on the legal standard that is in force. Such a full analysis is provided by David Friedman in "An Economic Analysis of Alternative Damage Rules for Breach of Contract," *Journal of Law and Economics* 23 (1989). Friedman establishes that either expectation or reliance damages could be more efficient, depending on circumstances.

Chapter 14

1c. The optimal outcome can be achieved with an entrance fee of 8 or 10 fish per day.

2d. She will charge 16 nuggets a day at Mine A and 21 nuggets a day at Mine B.

10. *Hint:* What happens to rental rates on the north side of town?

Chapter 15

4b. The short-run labor demand curve is less elastic than the long-run labor demand curve.

7. True. The reason why isocosts are straight lines is that their equations are given by $P_K \cdot K + P_L \cdot L = C$ where P_K, P_L, and C are constants. For a monopolist in the labor market, P_L is not constant: It varies with his employment of labor. Thus, the isocosts are not straight lines.

11. *Hint:* Graph the MP_L curve. Assume that the wage rate of labor rises from W to W'. Use your graph to illustrate the revenue earned by capital both before and after the wage change. Which is bigger?

12. *Hint:* Graph the MP_K curve. Assume that labor and capital are substitutes in production. Show how MP_K shifts in response to a rise in the wage rate of labor. Use your graph to illustrate the revenue earned by capital both before and after the wage change. Which is bigger? Is your answer consistent with your answer to problem 10? If not, what is the source of the discrepancy?

13. Apparently the union believes that a reduction in the quantity of unskilled labor (as would result from a minimum wage) would increase the demand for the skilled labor that its members supply. Thus, skilled and unskilled labor must be substitutes in production.

To investigate the relationship with capital, begin by dividing inputs into "unskilled labor" and "all other inputs," where the latter includes both skilled labor and capital. When there are zero profits and only two inputs, those inputs must be complements in production (this was shown in problem 10).

This means that a reduction in unskilled labor must reduce the demand for "all other inputs." Therefore, following a reduction in unskilled labor, the demand for either skilled labor or capital must fall. Because we have already agreed that the demand for skilled labor rises, it follows that the demand for capital falls. In other words, unskilled labor and capital are complements in production.

It follows that the owners of capital will oppose the minimum wage.

Chapter 16

2. Jack's budget line has a "kink" at 8 hours and is tangent to an indifference curve at 10 hours. Jill's budget line intersects the consumption axis at the same point as Jack's and is tangent to the same indifference curve. Drawing the picture, you will find that Jane's budget line must be steeper than the initial portion of Jack's but less steep than the later portion; in other words, W'' is between W and W'. The same picture should reveal that Jane works fewer than 10 hours.

4. *Hint:* Is leisure a normal or an inferior good for Dick?

9. *Hint:* Which of these men feels a greater income effect when his wage rate changes?

10. False. If workers come to enjoy their jobs, the supply curve of labor shifts out, the quantity supplied increases, and therefore the marginal product of labor decreases. So workers who enjoy their jobs more are *less* productive at the margin than those who enjoy them less.

11. The wage rate falls, less labor is supplied to the marketplace, and a given individual might supply either more or less labor than before.

13. The wage rate rises, less labor is supplied to the marketplace, and a given (surviving) individual supplies more labor than before.

17. One important difference arises from intertemporal substitution. In the circumstance of part (a), there is strong incentive to take one's vacation this year instead of next, whereas that incentive is missing from part (b). You should take account of this difference in determining the effects on wages and the quantity of labor supplied.

19. True, because education is a form of investment. Because the tax break applies to other forms of investment but not to education, investors tend to substitute toward those other forms of investment.

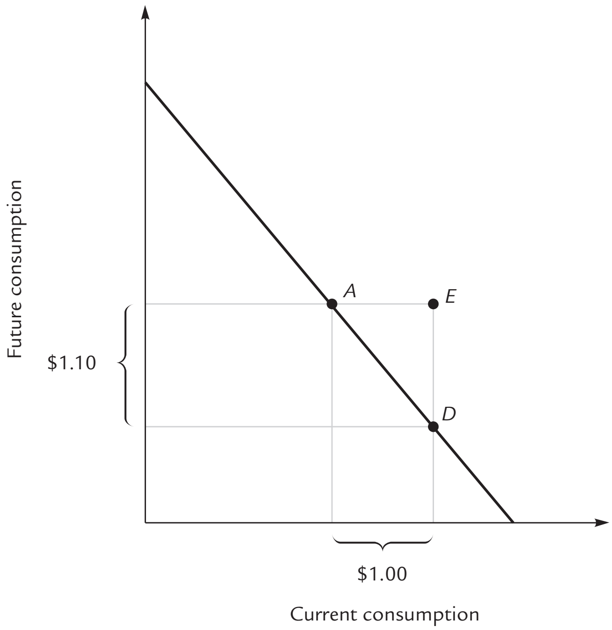
Chapter 17

5b. The halving of the interest rate is the greater deterrent.

12a. Jeeter can purchase a bond for $1,000 at a 10% interest rate and pretend that he has spent the $1,000 to pay off his loan. Five years from today, he simply hands the bond over to the bank. Because the bond and the debt grow at the same rate, the bond covers the debt exactly.

18. The interest rate is higher in the circumstance of (a).

21.



Terry starts with an endowment of A, faces an interest rate of 10%, and therefore has the pictured budget line with slope –1.10. If the government taxes him $1 and then provides him with $1 worth of current consumption, his endowment point remains A (nothing has really changed). If the government borrows $1 to provide Terry with $1 worth of current consumption, it then taxes him $1.10 in the future to repay the debt. Terry's endowment point shifts to D (with $1 more in present consumption and $1.10 less in future consumption). Because Terry's new endowment point is on his original budget line, his optimum consumption basket does not change. Each plan leads to the same demand for current consumption and so to the same equilibrium interest rate.

24. Here are a few observations:

First, Mr. Rohatyn asserts that borrowing will convert a $130 billion loss into a $500 billion drain over 20 or 30 years. In other words, he treats a dollar paid 20 years from now as equal in value to a dollar paid today. If he is really committed to such reasoning, Mr. Rohatyn should be happy to offer you a loan of $200 billion today in exchange for a payback of $300 billion in 20 years. Try writing to him and see if he agrees.

Second, he asserts that one should not borrow to finance losses that have already occurred, and elevates this dictum to "a basic economic principle." On the contrary, people generally prefer to spread out their consumption evenly over their lifetimes rather than having some years of feast and some of famine. (This is why we tend to think of the Great Depression as a bad thing.) It follows that a one-shot unexpected large expense is precisely the sort of thing that ought to be financed by borrowing. Your indifference curve analysis in part (b) should confirm this assertion.

Third, he is wrong in thinking that a short-term tax surcharge would necessarily limit the costs of the bailout to the immediate future. Precisely because people like to smooth out their consumption, they would borrow more (or, equivalently, save less) in the present to get through the temporary period of high taxes. The result would be the same as if the government had done the borrowing.

But not quite. For a variety of reasons, individuals must usually borrow at higher rates than the government does. Therefore, Mr. Rohatyn's proposal comes down to this: Let people attempt to borrow for themselves at high interest rates, rather than let the government borrow for them at lower rates.

Finally, some economists would argue that people are insufficiently sophisticated to borrow their way through the higher tax years (that is, some would argue that people fail to move to the optimum point in the indifference curve diagram). If those economists are right, then Mr. Rohatyn is even further off the mark, because these taxpayers in their naivete will fail to smooth out their consumption streams unless the government leads the way by borrowing for them.

26. $66,666.66

27. The interest rate rises.

Chapter 18

1. The desirability of the trade depends on the possible outcomes of the uncertainty and it depends on the odds. Had the doctor himself been offered the opportunity to trade a certain shilling for a mere 99% chance at a million pounds, he might have reconsidered his position. Indeed, to forgo suicide is to sacrifice the certainty of death for the uncertainties of life, but most of us make this "unwise" choice.

2. False. Even something that is not worth the certainty of death can still be worth some *chance* of death. If this were false, nobody would ever drive a car.

12. Much depends on whether higher crash rates are attributable to age or to lack of driving experience. The *Times* assumes that 18-year-old first-time drivers will crash at the same rate as 18-year-olds with 2 years' experience, which seems debatable.

Chapter 19

3. $\frac{U-C}{D-U} \cdot \left[\frac{D}{1+r} - S\right]$